

Spin relaxation and spin dynamics in semiconductors

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The spin of conduction electrons decays due to the combined effect of spin-orbit coupling and momentum scattering. The spin-orbit coupling couples the spin to the electron momentum, that is randomized by momentum scattering off of impurities and phonons. Seen from the perspective of the electron spin, the spin-orbit coupling gives a spin precession, while momentum scattering makes this precession randomly fluctuating, both in magnitude and orientation.

The specific mechanisms for the spin relaxation of conduction electrons were proposed by Elliott [1] and Yafet [2], for conductors with a center of inversion symmetry, and by D'yakonov and Perel' [3], for conductors without an inversion center. In p-doped semiconductors there is in play another spin relaxation mechanism, due to Bir, Aronov, and Pikus [4]. As this has a rather limited validity we do not describe it here. More details can be found in reviews [5, 6, 7, 8, 9].

Before we discuss the two main mechanisms, we introduce a toy model that captures the relevant physics of spin relaxation without resorting explicitly to quantum mechanics: *the electron spin in a randomly fluctuating magnetic field*. We will find certain universal qualitative features of the spin relaxation and dephasing in physically important situations.

The next part of this review covers the experimental as well as computational status of the field, discussing the spin relaxation in semiconductors under varying conditions such as temperature and doping density.

0.1 Toy model: the electron spin in a fluctuating magnetic field.

Consider an electron spin \mathbf{S} (or the corresponding magnetic moment) in the presence of an external time-independent magnetic field $\mathbf{B}_0 = B_0 \mathbf{z}$ giving rise to the Larmor precession frequency $\boldsymbol{\omega}_0 = \omega_0 \mathbf{z}$, and a fluctuating time-dependent field $\mathbf{B}(t)$ giving the Larmor frequency $\boldsymbol{\omega}(t)$; see Fig. 1. We assume that the field fluctuates about zero and is correlated on the time scale of τ_c :

$$\overline{\boldsymbol{\omega}(t)} = 0, \quad \overline{\omega_\alpha(t)\omega_\beta(t')} = \delta_{\alpha\beta}\overline{\omega_\alpha^2}e^{-|t-t'|/\tau_c}. \quad (1)$$

Here α and β denote the cartesian coordinates and the overline denotes averaging over different random realizations $\mathbf{B}(t)$. We will see later that such fluctuating fields arise quite naturally in the context of the electron spins in solids.

The following description applies equally to the classical magnetic moment described by the vector \mathbf{S} as well as to the quantum mechanical spin whose expectation value is \mathbf{S} . Writing out the torque equation, $\dot{\mathbf{S}} = \boldsymbol{\omega} \times \mathbf{S}$, we get the following equations of motion:

$$\dot{S}_x = -\omega_0 S_y + \omega_y(t) S_z - \omega_z(t) S_y, \quad (2)$$

$$\dot{S}_y = \omega_0 S_x - \omega_x(t) S_z + \omega_z(t) S_x, \quad (3)$$

$$\dot{S}_z = \omega_x(t) S_y - \omega_y(t) S_x. \quad (4)$$

These equations are valid for one specific realization of $\boldsymbol{\omega}(t)$. Our goal is to find instead effective equations for the time evolution of the average spin, $\overline{\mathbf{S}}(t)$, given the ensemble of Larmor frequencies $\boldsymbol{\omega}(t)$.

0.1. TOY MODEL: THE ELECTRON SPIN IN A FLUCTUATING MAGNETIC FIELD. 3

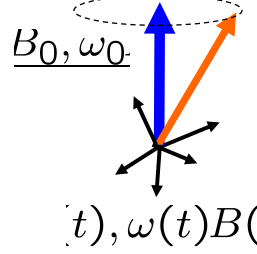


Figure 1: Electron spin precesses about the static B_0 field along \mathbf{z} . The randomly fluctuating magnetic field $\mathbf{B}(t)$ causes spin relaxation and spin dephasing.

It is convenient to introduce the complex “rotating” spins S_{\pm} and Larmor frequencies ω_{\pm} in the (x, y) plane:

$$S_+ = S_x + iS_y, \quad S_- = S_x - iS_y, \quad (5)$$

$$\omega_+ = \omega_x + i\omega_y, \quad \omega_- = \omega_x - i\omega_y. \quad (6)$$

The inverse relations are

$$S_x = \frac{1}{2}(S_+ + S_-), \quad S_y = \frac{1}{2i}(S_+ - S_-), \quad (7)$$

$$\omega_x = \frac{1}{2}(\omega_+ + \omega_-), \quad \omega_y = \frac{1}{2i}(\omega_+ - \omega_-). \quad (8)$$

The equations of motion for the spin set (S_+, S_-, S_z) are,

$$\dot{S}_+ = i\omega_0 S_+ + i\omega_z S_+ - i\omega_+ S_z, \quad (9)$$

$$\dot{S}_- = -i\omega_0 S_- - i\omega_z S_- + i\omega_- S_z, \quad (10)$$

$$\dot{S}_z = -(1/2i)(\omega_+ S_- - \omega_- S_+). \quad (11)$$

In the absence of the fluctuating fields the spin S_+ rotates in the complex plane anticlockwise (for $\omega_0 > 0$), while S_- clockwise.

The precession about B_0 can be factored out by applying the ansatz,¹:

$$S_{\pm} = s_{\pm}(t)e^{\pm i\omega_0 t}. \quad (12)$$

Indeed, it is straightforward to find the time evolution of the set $(s_+, s_-, s_z \equiv S_z)$:

$$\dot{s}_+ = i\omega_z s_+ - i\omega_+ s_z e^{-i\omega_0 t}, \quad (13)$$

$$\dot{s}_- = -i\omega_z s_- + i\omega_- s_z e^{i\omega_0 t}, \quad (14)$$

$$\dot{s}_z = -(1/2i)(\omega_+ s_- e^{-i\omega_0 t} - \omega_- s_+ e^{i\omega_0 t}). \quad (15)$$

The penalty for transforming into this “rotating frame” is the appearance of the phase factors $\exp(\pm i\omega_0 t)$.

¹This is analogous to going to the interaction picture when dealing with a quantum mechanical problem of that type.

The solutions of Eqs. 13, 14, and 15, can be written in terms of the integral equations,

$$s_+(t) = s_+(0) + i \int_0^t dt' \omega_z(t') s_+(t') - i \int_0^t dt' \omega_+(t') s_z(t') e^{-i\omega_0 t'}, \quad (16)$$

$$s_-(t) = s_-(0) - i \int_0^t dt' \omega_z(t') s_-(t') + i \int_0^t dt' \omega_-(t') s_z(t') e^{i\omega_0 t'}, \quad (17)$$

$$s_z(t) = s_z(0) - \frac{1}{2i} \int_0^t dt' \left[\omega_+(t') s_-(t') e^{-i\omega_0 t'} - \omega_-(t') s_+(t') e^{i\omega_0 t'} \right]. \quad (18)$$

We should now substitute the above solutions back into Eqs. 13, 14, and 15. The corresponding expressions become rather lengthy, so we demonstrate the procedure on the s_+ component only. We get,

$$\begin{aligned} \dot{s}_+(t) = & i\omega_z(t)s_+(0) - \omega_z(t) \int_0^t dt' \omega_z(t') s_+(t') + \omega_z(t) \int_0^t dt' \omega_+(t') s_z(t') e^{-i\omega_0 t'} \\ & - i\omega_+(t) e^{-i\omega_0 t} s_z(0) + \frac{1}{2} e^{-i\omega_0 t} \omega_+(t) \int_0^t dt' \left[\omega_+(t') s_-(t') e^{-i\omega_0 t'} - \omega_-(t') s_+(t') e^{i\omega_0 t'} \right] \end{aligned} \quad (19)$$

The reader is encouraged to write the analogous equations for \dot{s}_- (that for \dot{s}_- is easy to write since $s_- = s_+^*$).

We now make two approximations. First, we assume that the fluctuating field is rather weak and stay in *the second order* in ω .² This allows us to factorize the averaging over the statistical realizations of the field,

$$\overline{\omega(t)\omega(t')s(t')} \approx \overline{\omega(t)\omega(t')} \overline{s(t')} \quad (20)$$

as the spin changes only weakly over the time scale, τ_c , of the changes of the fluctuating fields. This approximation is called the *Born approximation*, alluding to the analogy with the second-order time-dependent perturbation theory in quantum mechanics. Going beyond the Born approximation one would need to execute complicated averaging schemes of the product in Eq. 20, since $s(t)$ in general depends on $\omega(t' \leq t)$.

As the second assumption, we consider a “coarse-grained” time evolution, meaning that we are effectively averaging $s(t)$ over the time scale of the correlation time τ_c ; we are interested in times t much greater than τ_c . That allows us to approximate,

$$\int_0^{t \gg \tau_c} dt' \overline{\omega(t)\omega(t')} \overline{s(t')} \approx \int_0^{t \gg \tau_c} dt' \overline{\omega(t)\omega(t')} \overline{s(t)}, \quad (21)$$

since the correlation function $\overline{\omega(t)\omega(t')}$ is significant in the time interval of $|t - t'| \approx \tau_c$ only. The above approximation makes clear that the spin $s(t)$ is the representative coarse-grained (running-averaged) spin of the time interval $(t - \tau_c, t)$. Equation 21 is a realization of the *Markov approximation*. The physical meaning is that the spin s varies only slowly on the time scale of τ_c over which the correlation of the fluctuating fields is significant. We then need to restrict ourselves to the time scales t larger than the correlation time τ_c . In effect, we will see that in this approximation *the rate of change of the spin at a given time depends on the spin at that time, not on the previous history of the spin.*

²More precisely, we assume that $|\omega(t)|\tau_c \ll 1$, so that the spin does not fully precess about the fluctuating field before the field makes a random change.

0.1. TOY MODEL: THE ELECTRON SPIN IN A FLUCTUATING MAGNETIC FIELD. 5

Applying the Born-Markov approximation to Eq. 19 we obtain for the average spin \bar{s}_+ the following time evolution equation:³

$$\begin{aligned} \dot{\bar{s}}_+ = & \overline{i\omega_z(t)s_+(0)} - \int_0^t dt' \overline{\omega_z(t)\omega_z(t')} \overline{s_+(t)} + \int_0^t dt' \overline{\omega_z(t)\omega_+(t')} e^{-i\omega_0 t'} \overline{s_z(t)} \\ & - \overline{i\omega_+(t)} e^{-i\omega_0 t} s_z(0) + \frac{1}{2} e^{-i\omega_0 t} \int_0^t dt' \left[\overline{\omega_+(t)\omega_+(t')} e^{-i\omega_0 t'} \overline{s_-(t)} - \overline{\omega_+(t)\omega_-(t')} e^{i\omega_0 t'} \overline{s_+(t)} \right] \end{aligned} \quad (22)$$

Using the rules of Eqs. 1 the above simplifies to

$$\dot{\bar{s}}_+ = -\overline{\omega_z^2} \int_0^t dt' e^{-(t-t')/\tau_c} \overline{s_+(t)} + \frac{1}{2} e^{-i\omega_0 t} \int_0^t dt' \left[(\overline{\omega_x^2} - \overline{\omega_y^2}) e^{-i\omega_0 t'} \overline{s_-(t)} - (\overline{\omega_x^2} + \overline{\omega_y^2}) e^{i\omega_0 t'} \overline{s_+(t)} \right] e^{-(t-t')/\tau_c}. \quad (23)$$

Since we consider the times $t \gg \tau_c$, we can approximate

$$\int_0^t dt' e^{-(t-t')/\tau_c} \approx \int_{-\infty}^t dt' e^{-(t-t')/\tau_c} = \tau_c. \quad (24)$$

Similarly,

$$\int_0^t dt' e^{-(t-t')/\tau_c} e^{-i\omega_0(t \pm t')} \approx \int_{-\infty}^t dt' e^{-(t-t')/\tau_c} e^{-i\omega_0(t \pm t')} = \tau_c \frac{1 \mp i\omega_0 \tau_c}{1 + \omega_0^2 \tau_c^2}. \quad (25)$$

The imaginary parts induce the precession of s_{\pm} , which is equivalent to shifting (renormalizing) the Larmor frequency ω_0 . The relative change of the frequency is $(\omega\tau_c)^2$ which is assumed much smaller than one by our Born approximation. We thus keep the real parts only and obtain,

$$\dot{\bar{s}}_+ = -\overline{\omega_z^2} \tau_c \overline{s_+} + \frac{1}{2} \frac{\tau_c}{1 + \omega_0^2 \tau_c^2} \left[(\overline{\omega_x^2} - \overline{\omega_y^2}) \overline{s_-} e^{-2i\omega_0 t} - (\overline{\omega_x^2} + \overline{\omega_y^2}) \overline{s_+} \right]. \quad (26)$$

Using the same procedure (or simply using $s_- = s_+^*$) we would arrive for the analogous equation for s_- :

$$\dot{\bar{s}}_- = -\overline{\omega_z^2} \tau_c \overline{s_-} + \frac{1}{2} \frac{\tau_c}{1 + \omega_0^2 \tau_c^2} \left[(\overline{\omega_x^2} - \overline{\omega_y^2}) \overline{s_+} e^{2i\omega_0 t} - (\overline{\omega_x^2} + \overline{\omega_y^2}) \overline{s_-} \right]. \quad (27)$$

Similarly,

$$\dot{\bar{s}}_z = -(\overline{\omega_x^2} + \overline{\omega_y^2}) \frac{\tau_c}{1 + \omega_0^2 \tau_c^2} \overline{s_z} \quad (28)$$

For the rest of the section we omit the overline on the symbols for the spins, so that S will mean the average spin. Returning back to our rest frame of the spins rotating with frequency ω_0 , we get

$$\dot{S}_+ = i\omega_0 S_+ - \overline{\omega_z^2} \tau_c \frac{1}{2} \frac{\tau_c}{1 + \omega_0^2 \tau_c^2} \left[(\overline{\omega_x^2} - \overline{\omega_y^2}) S_- - (\overline{\omega_x^2} + \overline{\omega_y^2}) S_+ \right], \quad (29)$$

$$\dot{S}_- = i\omega_0 S_- - \overline{\omega_z^2} - \frac{1}{2} \frac{\tau_c}{1 + \omega_0^2 \tau_c^2} \left[(\overline{\omega_x^2} - \overline{\omega_y^2}) S_+ - (\overline{\omega_x^2} + \overline{\omega_y^2}) S_- \right], \quad (30)$$

$$\dot{S}_z = -(\overline{\omega_x^2} + \overline{\omega_y^2}) \frac{\tau_c}{1 + \omega_0^2 \tau_c^2} \overline{S_z}. \quad (31)$$

³The initial values of the spin, $s(0)$, are fixed and not affected by averaging.

Finally, going back to S_x and S_y :

$$\dot{S}_x = -\omega_0 S_y - \overline{\omega_z^2} \tau_c S_x - \frac{\tau_c}{1 + \omega_0^2 \tau_c^2} \overline{\omega_y^2} S_x \quad (32)$$

$$\dot{S}_y = \omega_0 S_x - \overline{\omega_z^2} \tau_c S_y - \frac{\tau_c}{1 + \omega_0^2 \tau_c^2} (\overline{\omega_x^2}) S_y, \quad (33)$$

$$\dot{S}_z = -(\overline{\omega_x^2} + \overline{\omega_y^2}) \frac{\tau_c}{1 + \omega_0^2 \tau_c^2} S_z. \quad (34)$$

We can give the above equation a more conventional form, by introducing two types of the spin decay times. First, we define the *spin relaxation time* T_1 by,

$$\frac{1}{T_1} = (\overline{\omega_x^2} + \overline{\omega_y^2}) \frac{\tau_c}{1 + \omega_0^2 \tau_c^2}, \quad (35)$$

and the spin dephasing times T_2 by

$$\frac{1}{T_{2x}} = \overline{\omega_z^2} \tau_c + \frac{\overline{\omega_y^2} \tau_c}{1 + \omega_0^2 \tau_c^2}, \quad (36)$$

$$\frac{1}{T_{2y}} = \overline{\omega_z^2} \tau_c + \frac{\overline{\omega_x^2} \tau_c}{1 + \omega_0^2 \tau_c^2}. \quad (37)$$

We then write:

$$\dot{S}_x = -\omega_0 S_y - \frac{S_x}{T_{2x}}, \quad (38)$$

$$\dot{S}_y = \omega_0 S_x - \frac{S_y}{T_{2y}}, \quad (39)$$

$$\dot{S}_z = -\frac{S_z}{T_1}. \quad (40)$$

Our fluctuating field is effectively at infinite temperature, at which the average value for the spin in a magnetic field is zero. A more general spin dynamics is

$$\dot{S}_x = -\omega_0 S_y - \frac{S_x}{T_{2x}}, \quad (41)$$

$$\dot{S}_y = \omega_0 S_x - \frac{S_y}{T_{2y}}, \quad (42)$$

$$\dot{S}_z = -\frac{S_z - S_{0z}}{T_1}. \quad (43)$$

where S_{0z} is the equilibrium value of the spin in the presence of the static magnetic field of the Larmor frequency ω_0 at the temperature at which the environmental fields giving rise to $\omega(t)$ are in equilibrium. The above equations are called the *Bloch equations*.

The spin components S_x and S_y , which are perpendicular to the applied static field \mathbf{B}_0 , decay exponentially on the time scales of T_{2x} and T_{2y} , respectively. These times are termed spin *dephasing* times, as they describe the loss of the *phase* of the spin components perpendicular to the static field \mathbf{B}_0 . They are also often called *transverse* times, for that reason.

0.1. TOY MODEL: THE ELECTRON SPIN IN A FLUCTUATING MAGNETIC FIELD. 7

The time T_1 is termed the spin *relaxation* time, as it describes the (thermal) relaxation of the spin to the equilibrium. During the spin relaxation in a static magnetic field the energy is exchanged with the environment. In the language of statistical physics, the relaxation process establishes the Boltzmann probability distribution for the system. Similarly, dephasing establishes the “random phases” postulate that says that there is no correlation (coherence) among the degenerate states, such as the two transverse spin orientations; in thermal equilibrium such states add incoherently.

For the sake of discussion consider an isotropic system in which

$$\overline{\omega_x^2} = \overline{\omega_y^2} = \overline{\omega_z^2} = \overline{\omega^2}. \quad (44)$$

If the static magnetic field is weak, $\omega_0\tau_c \ll 1$, the three times are equal:

$$T_1 = T_{2x} = T_{2y} = \frac{1}{\overline{\omega^2}\tau_c}. \quad (45)$$

There is no difference between the spin relaxation and spin dephasing. It is at first sight surprising that the spin relaxation time is inversely proportional to the correlation time. The more random the external field appears, the less the spin decays. We will explain this fact below by the phenomenon of motional narrowing.

In the opposite limit of large Larmor frequency, $\omega_0\tau_c \gg 1$, the spin relaxation rate vanishes,

$$\frac{1}{T_1} \approx \frac{\overline{\omega^2}}{\omega_0^2} \frac{1}{\tau_c} \rightarrow 0, \quad (46)$$

while the spin dephasing time is given by what is called *secular broadening*,

$$\frac{1}{T_2} \approx \overline{\omega_z^2}\tau_c. \quad (47)$$

If secular broadening is absent, the leading term in the dephasing time will be, as in the relaxation,

$$\frac{1}{T_2} \approx \frac{\overline{\omega^2}}{\omega_0^2} \frac{1}{\tau_c}. \quad (48)$$

In this limit the spin dephasing rate is proportional to the correlation rate, not to the correlation time.

In the cases in which there is no clear distinction between T_1 and T_2 , we often use the symbol

$$\tau_s = T_1 = T_2, \quad (49)$$

to describe the *generic spin relaxation*.

Motional narrowing

The surprising fact that, at low magnetic fields, the spin relaxation rate is proportional to the correlation time of the fluctuating field (as opposed to its inverse), is explained by *motional narrowing*. Consider the spin transverse to an applied magnetic field and assume

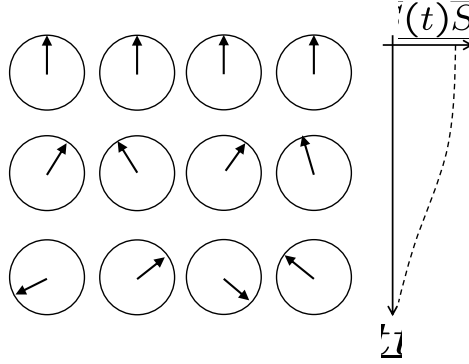


Figure 2: Electron spin precesses about the static \mathbf{B}_0 field along \mathbf{z} perpendicular to this page. The spatially fluctuating magnetic field $B_1\mathbf{z}$ causes reversible spin spin dephasing.

that the field has a single magnitude, but can randomly switch directions, between up to down, leading to a random precession of the spin clock and anticlockwise. In effect, the spin phase executes a random walk. A single step takes the time τ_c , the correlation time of the fluctuating field. After n steps, that is, after the time $t = n\tau$, the standard deviation of the phase will be $\delta\phi = (\omega\tau_c)\sqrt{n}$, the well known result for a random walk. We call the spin dephasing time the time it takes for $\delta\phi \approx 1$. This happens after the time $\tau_s = \tau_c/(\omega\tau_c)^2$, or $\tau_s = 1/(\omega^2\tau_c)$, which is the result we obtained earlier from the Born-Markov approximation, Eq. 45.

0.1.1 Reversible dephasing, spin ensemble, random walk in inhomogeneous fields.

Our previous calculation was carried out for a single spin in a fluctuating magnetic field. After the decay of the spin components, the information about the original spin is irreversibly lost as we have no information on the actual history of the fluctuating field. Such an irreversible loss of spin is often termed spin *decoherence*. We will see that spin dephasing can be reversible. We will also see that a simple exponential decay, of the type $\exp(-t/\tau_s)$, is not a rule.

Reversible spin dephasing: spin ensemble in spatially random magnetic field

There are physically relevant cases in which the decay of spin is reversible. A typical example is an *ensemble* of localized spins, each precessing about a static local magnetic field $\mathbf{B}_0 + \mathbf{B}_1$, with varying static components \mathbf{B}_1 (of zero average) giving rise to random precession frequencies ω_1 . See Fig. 2. Another important example is that of the conduction electrons in noncentrosymmetric crystals, such as GaAs, in which the spin-orbit coupling acts as a momentum dependent magnetic field. The spins of the electrons of different momenta precess with different frequencies. We are interested in the total spin as the sum of the individual spins.

Consider the external field along z direction, and take the fluctuating frequencies from

0.1. TOY MODEL: THE ELECTRON SPIN IN A FLUCTUATING MAGNETIC FIELD. 9

the Gaussian distribution:

$$P(\omega_1) = \frac{1}{\sqrt{2\pi\delta\omega^2}} e^{-\omega_1^2/2\delta\omega^2}, \quad (50)$$

with zero mean and $\delta\omega_1^2$ variance. Denote the in-plane spin of the electron a ⁴ by S_x^a and S_y^a . This spin precesses with the frequency $\omega_0 + \omega_1^a$, leading to the time evolution for the rotating spins

$$S_{\pm}^a(t) = S_{\pm}^a(0) e^{\pm i\omega_0 t} e^{\pm i\omega_1^a t}. \quad (51)$$

Suppose at $t = 0$ all the spins are lined up, that is, $S_{\pm}^a(0) = S_{\pm}(0)$. The total spin $S_{\pm}(t)$ is the sum,

$$S_{\pm}(t) = \sum_s S_{\pm}^a(t) = S_{\pm}(0) e^{\pm i\omega_0 t} \int_{-\infty}^{\infty} d\omega_1 P(\omega_1) e^{\pm i\omega_1 t}. \quad (52)$$

Evaluating the Gaussian integral we get

$$S_{\pm}(t) = S_{\pm}(0) e^{\pm i\omega_0 t} e^{-\delta\omega_1^2 t^2/2}. \quad (53)$$

The in-plane component vanishes after the time of about $1/\delta\omega_1$, but this dephasing of the spin is reversible, since each individual spin preserves the memory of the initial state. The disappearance of the spin is purely due to the statistical averaging over an ensemble in which the individual spins have, after certain time, random phases. This spin decay is not a simple exponential, but rather Gaussian.

Spin echo

We have seen that there are irreversible and reversible effects both present in spin dephasing. It turns out that the reversible effects can be separated out by the phenomenon of the spin echo. Figure 3 explains this mechanism in detail. Suppose all the localized spins in our ensemble point in one direction at time $t = 0$. At a later time, $t = T_{\pi}$, the spins dephase due to the inhomogeneities of the precession frequencies and the total spin is small. We apply a short pulse of an external magnetic field, the so called π pulse, that rotates the spins along the axis parallel to the original spin direction, mapping the spins as $(S_x, S_y) \rightarrow (-S_x, S_y)$. At that moment the spins will still be dephased, but the one that is the fastest is now the last, and the one that is the slowest, appears as the first. At the time $t = 2T_{\pi}$ all the spins catch on, producing a large spin signal along the original spin direction. In reality this signal will be weaker than that at $t = 0$ due to the presence of irreversible processes, by $\exp(-2T_{\pi}/T_2)$. Important, reversible processes are not counted in T_2 .

Spin random walk in inhomogeneous magnetic field

Another interesting situation appears when we consider the possibility that the spin diffuses through a region of an inhomogeneous precession frequency.⁵ We can model this situation on the system of spins in one dimension, each spin jumping in a time τ left or right. Suppose the precession frequencies vary in the x direction as

$$\omega(x) = \omega_0 + \omega'x. \quad (54)$$

⁴This discussions applies to nuclear spins as well.

⁵The inhomogeneity could be due to the magnetic field or to the g-factor.

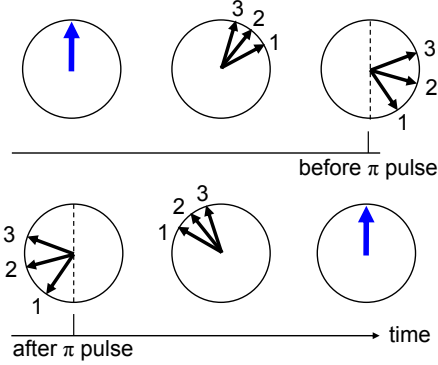


Figure 3: Initially all the spins point up. Due to random precession frequencies the spins soon point in different directions and the average spin vanishes. Applying a π -pulse rotating the spins along the vertical axis makes the fastest spin the slowest, and vice versa. After the fastest ones catch up again with the slowest, the original value of the average spin is restored. Any reduction from the original value is due to irreversible processes. signal.

Here ω' is the gradient of ω . As the electron diffuses, it precesses with the precession frequency $\omega(t) = \omega[x(t)]$, given by the position of the electron $x(t)$ at time t . The time evolution for an individual spin is

$$\dot{S}_+ = \omega(t)S_+(t) = \omega(t)S_+(0) + \omega(t) \int_0^\infty dt' \omega(t')S(t'). \quad (55)$$

We are interested in the averaged quantities over many realizations of the random walk. We also consider the time scales much greater than the individual random walks steps τ .

The accumulated phase after N steps or $t = N\tau$ time is the sum

$$\phi_N = x_1 + x_2 + \dots + x_N = N\delta_1 + (N-1)\delta_1 + \dots + \delta_N. \quad (56)$$

Here $\delta_i = \pm 1$ is a random variable representing the random step left or right. The variance of ϕ_N is

$$\sigma_N^2 = \sum_i^{N-1} (N-i)^2 \approx \frac{1}{3}N^3. \quad (57)$$

According to the central limit theorem, ϕ_N is distributed normally with the above variance:

$$P(\phi_N) = \frac{1}{\sqrt{2\pi\sigma_N^2}} e^{-\phi_N^2/2\sigma_N^2}. \quad (58)$$

We can now transform the dynamical equation into the ensemble averaging:

$$S_+(t) = S_+(0)e^{i\phi(t)} = \int_{-\infty}^{\infty} d\phi_N e^{i\phi(t)} P(\phi_N) = e^{-\omega_1^2 D t^3/3}, \quad (59)$$

where we denoted the diffusivity by $D = \tau/2$, for the unit length step.

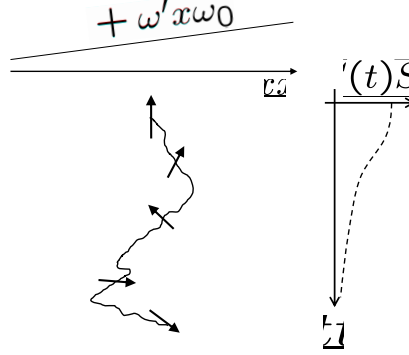


Figure 4: The electron performs a random walk. Its spin precesses by the inhomogeneous magnetic field along the x axis. The average spin dephases to zero with time.

0.1.2 Quantum mechanical description

The Born-Markov approximation to the dynamics of a system coupled to an external environment can be cast in the quantum mechanical language. We refer the reader to Ref. [8] for the derivation. The physics of this derivation is the same as what we did in Sec. 0.1. Only the formalism is different.

We consider the system described by the Hamiltonian

$$H(t) = H_0 + V(t), \quad (60)$$

in which H_0 is our system *per se* and $V(t)$ is the time-dependent random fluctuating field, of zero average and correlation time τ_c :

$$\overline{V(t)} = \overline{V(t)} = 0, \quad \overline{V(t)V(t')} \sim e^{-|t-t'|/\tau_c}. \quad (61)$$

The system is fully described by the density matrix ρ .

The transformation to the rotating frame is equivalent to going to the interaction picture in quantum mechanics:

$$\rho_I(t) = e^{iH_0 t/\hbar} \rho e^{-iH_0 t/\hbar}, \quad (62)$$

$$V_I(t) = e^{iH_0 t/\hbar} V(t) e^{-iH_0 t/\hbar}. \quad (63)$$

Performing the operations as outlined in Sec. 0.1 for the classical model, we arrive at the effective time evolution for the density of state operator,

$$\frac{d\overline{\rho_I(t)}}{dt} = \left(\frac{1}{i\hbar}\right)^2 \int_0^{t \gg \tau_c} dt' \left[\overline{V_I(t), [V_I(t'), \rho_I(t)]} \right]. \quad (64)$$

The above equation is called the *Master equation* and is the starting equation in many important problems in which a quantum system is in contact with a reservoir.

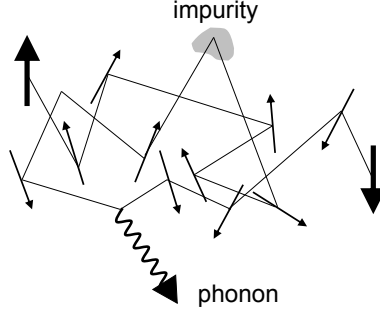


Figure 5: *D'yakonov-Perel' mechanism*. The electron starts with the spin up. As it moves, its spin precesses about the axis corresponding to the electron velocity. Phonons and impurities change the velocity, making the spin to precess about a different axis (and with different speed). During the scattering event the spin is preserved.

0.1.3 Spin relaxation of conduction electrons

We will consider nonmagnetic conductors in zero or weak magnetic fields so that the spin dephasing and spin relaxation times are equal, $T_2 = T_1 = \tau_s$. The formula for the spin relaxation time,

$$1/\tau_s = \omega^2 \tau_c, \quad (65)$$

that was derived above for the spin in a fluctuating magnetic field, applies in a semiquantitative sense (that is, it gives an order of magnitude estimates and useful trends) to the conduction electron spins as well. We analyze below the D'yakonov-Perel'[3] and the Elliott-Yafet[1, 2] mechanisms.

The D'yakonov-Perel' mechanism is at play in solids lacking a center of spatial inversion symmetry. The most prominent example is the semiconductor GaAs. In such solids the spin-orbit coupling is manifested as some effective magnetic field—the *spin-orbit field*—that depends on the electron momentum. Electrons in different momentum states feel different spin-orbit fields, so that the spin precesses with a given Larmor frequency, until the electron is scattered into another momentum state. See Fig. 5. As the electron momentum changes on the time scale τ of the momentum relaxation time, the net effect of the momentum scattering on the spin is to produce random fluctuations of the Larmor frequencies. We have motional narrowing. Since these frequencies are correlated by $\tau_c = \tau$, we arrive at

$$\frac{1}{\tau_s} = \omega_{so}^2 \tau, \quad (66)$$

for the spin relaxation time. The magnitude of ω_{so} is the measure of the strength of the spin-orbit coupling. The spin relaxation rate is directly proportional to the momentum relaxation time: the more the electron scatters, the less its spin dephases.

The Elliott-Yafet mechanism works in systems with and without a center of inversion. It relies on spin-flip momentum scattering. The spin-flip amplitudes are due to spin-orbit

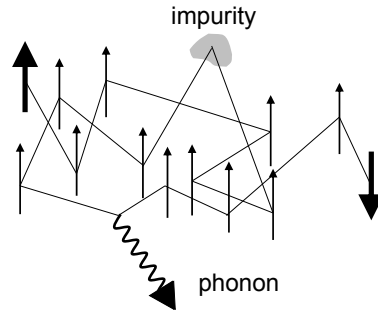


Figure 6: *Elliott-Yafet mechanism*. The electron starts with the spin up. As it scatters off of impurities and phonons, the spin can also flip due to spin-orbit coupling. Between the scattering the spin is preserved. After, say, a million scattering events, the spin will be down.

coupling, while the momentum scattering is due to the presence of impurities (that also contribute to the spin-orbit coupling), phonons, rough boundaries, or whatever is capable of randomizing the electron momentum. During the scattering events, the spin is preserved. See Fig. 6. How do we account for such a scenario with our toy model? Consider an electron scattering off of an impurity with a spin flip. This spin flip can be viewed as a precession that occurs during the time of the interaction of the electron with the impurity. Let us say that the scattering takes the time of λ_F/v_F , where v_F is the electron velocity and λ_F is the electron wave-length at the Fermi level (considering that it is greater or at most equal to the size a of the impurity—otherwise we could equally put a/v_F). Then the precession angle $\varphi = \omega_{so}(\lambda_F/v_F)$, with ω_{so} denoting the spin-orbit coupling induced precession frequency. Let us compare this with the angle of precession, $\omega\tau$, in the motional narrowing model, Eq. 65. We see that the spin-flip can be described by the effective precession frequency $\omega = \omega_{so}(\lambda_F/v_F\tau)$. We then obtain for the spin relaxation rate

$$\frac{1}{\tau_s} = \omega_{so}^2 \frac{\lambda_F^2}{v_F^2 \tau} \approx \left(\frac{\varepsilon_{so}}{\varepsilon_F} \right)^2 \frac{1}{\tau}. \quad (67)$$

Here $\varepsilon_{so} = \hbar\omega_{so}$ and $\varepsilon = (\hbar k_F)v_F/2$ is the Fermi energy. For the the Elliott-Yafet mechanism holds that the more the electron scatters, the more the spin dephases.

Below we discuss the two mechanisms in more detail, providing the formalisms for their investigation.

0.2 The D'yakonov-Perel' mechanism

D'yakonov and Perel' [3] considered solids without a center of inversion symmetry, such as GaAs or InAs. The mechanism also works for electrons at surfaces and interfaces. In such solids the presence of spin-orbit coupling induces the spin-orbit fields which give rise to the spin precession. Momentum relaxation then causes the random walk of the spin phases.

Spin-orbit field

In solids without a center of inversion symmetry, spin-orbit coupling splits the electron energies:

$$\varepsilon_{\mathbf{k},\uparrow} \neq \varepsilon_{\mathbf{k},\downarrow}. \quad (68)$$

Only the Kramers degeneracy is left, due to time reversal symmetry:

$$\varepsilon_{\mathbf{k},\uparrow} = \varepsilon_{-\mathbf{k},\downarrow}. \quad (69)$$

This energy splitting at a given momentum \mathbf{k} is conveniently described by the spin-orbit field $\mathbf{\Omega}$, giving a Zeeman-like (but momentum dependent) energy contribution to the electronic states, described by the additional Hamiltonian (to the usual band structure):

$$H_1 = \frac{\hbar}{2} \mathbf{\Omega}_{\mathbf{k}} \cdot \boldsymbol{\sigma}. \quad (70)$$

The time reversal symmetry requires that the spin-orbit field is an odd function of the momentum:

$$\mathbf{\Omega}_{\mathbf{k}} = -\mathbf{\Omega}_{-\mathbf{k}}. \quad (71)$$

The representative example of a spin-orbit field is the Bychkov-Rashba [10] (α_{BR}) and Dresselhaus [11, 12] (γ_D) couplings in 2d electron gases formed at the zinc-blende heterostructures grown along [001] [8]:

$$H_1 = (\alpha_{BR} + \gamma_D)\sigma_x k_y - (\alpha_{BR} - \gamma_D)\sigma_y k_x. \quad (72)$$

The axes are $\mathbf{x} = [110]$ and $y = [1\bar{1}0]$. The spin-orbit field

$$\hbar \mathbf{\Omega}_{\mathbf{k}} = 2 [(\alpha_{BR} + \gamma_D)k_y, -(\alpha_{BR} - \gamma_D)k_x]. \quad (73)$$

This field has the C_{2v} symmetry, reflecting the structural symmetry of the zinc-blend interfaces (such as GaAs/GaAlAs) with the principal axes along $[110]$ and $[1\bar{1}0]$.

Kinetic equation for the spin

Let $\mathbf{s}_{\mathbf{k}}$ be the electron spin in the momentum state \mathbf{k} . The time evolution of the spin is then described by

$$\frac{\partial \mathbf{s}_{\mathbf{k}}}{\partial t} - \mathbf{\Omega}_{\mathbf{k}} \times \mathbf{s}_{\mathbf{k}} = - \sum_{\mathbf{k}'} W_{\mathbf{k}\mathbf{k}'} (\mathbf{s}_{\mathbf{k}} - \mathbf{s}_{\mathbf{k}'}'). \quad (74)$$

The left hand side gives the full time derivative $d\mathbf{s}_{\mathbf{k}}/dt$, which is due to the explicit change of the spin, its direction, and the change of the spin due to the change of the momentum \mathbf{k} . The right-hand side is the change of the spin at \mathbf{k} due to the spin-preserving scattering from and to that state. The scattering rate between the two momentum states \mathbf{k} and \mathbf{k}' is $W_{\mathbf{k}\mathbf{k}'}$.

There are two time scales in the problem. First, momentum scattering, which occurs on the time scale of the momentum relaxation time τ , makes the spins in different momentum states equal. Second, this uniform spin decays on the time scale of the spin relaxation time τ_s which we need to find. We assume that $\tau \ll \tau_s$; this assumption is well satisfied in real systems. In principle we could go directly to our model of the electron spin in a fluctuating

magnetic field, with the role of the random Larmor precession playing by Ω , identifying $\tau_c = \tau$. However, it is instructive to see how this two time-scale problem is solved directly.

We separate the fast and slow components of the spins,

$$\mathbf{s}_{\mathbf{k}} = \mathbf{s} + \boldsymbol{\xi}_{\mathbf{k}}, \quad \langle \boldsymbol{\xi}_{\mathbf{k}} \rangle = 0. \quad (75)$$

The symbol $\langle \dots \rangle$ denotes averaging over different momenta. Our goal is to find the effective equation for the time evolution of \mathbf{s} , which is the actual spin averaged over \mathbf{k} . The fast component, $\boldsymbol{\xi}_{\mathbf{k}}$, decays on the time scale of τ to the value given by the instantaneous value of \mathbf{s} . Our goal is to find the effective equation for the time evolution of \mathbf{s} , which is the actual spin averaged over \mathbf{k} . The fast component, $\boldsymbol{\xi}_{\mathbf{k}}$, decays on the time scale of τ to the quasistatic value given by the instantaneous value of \mathbf{s} .

Upon substituting Eq. 75 into the kinetic equation Eq. 74 and averaging over \mathbf{k} , we obtain the equation of motion for the averaged spin

$$\dot{\mathbf{s}} = \langle \Omega_{\mathbf{k}} \times \boldsymbol{\xi}_{\mathbf{k}} \rangle \quad (76)$$

using that $\langle \Omega_{\mathbf{k}} \rangle = 0$. We need to find $\boldsymbol{\xi}_{\mathbf{k}}$. Since \mathbf{s} is hardly changing on the time scales relevant to $\boldsymbol{\xi}_{\mathbf{k}}$, we write,

$$\dot{\boldsymbol{\xi}}_{\mathbf{k}} - \Omega_{\mathbf{k}} \times \mathbf{s} - \Omega_{\mathbf{k}} \times \boldsymbol{\xi}_{\mathbf{k}} = - \sum_{\mathbf{k}'} W_{\mathbf{k}\mathbf{k}'} (\boldsymbol{\xi}_{\mathbf{k}} - \boldsymbol{\xi}_{\mathbf{k}'}). \quad (77)$$

We make the following assumption:

$$\Omega\tau \ll 1. \quad (78)$$

That is, we assume that the precession is slow on the time scale of the momentum relaxation time (see our note on the Born approximation in the toy model section Sec. 0.1. For simplicity, we make the relaxation time approximation to model the time evolution of the fast component $\boldsymbol{\xi}_{\mathbf{k}}$:

$$\dot{\boldsymbol{\xi}}_{\mathbf{k}} - \Omega_{\mathbf{k}} \times \mathbf{s} - \Omega_{\mathbf{k}} \times \boldsymbol{\xi}_{\mathbf{k}} = - \frac{\boldsymbol{\xi}_{\mathbf{k}}}{\tau}. \quad (79)$$

From the condition of the quasistatic behavior, $\partial \boldsymbol{\xi}_{\mathbf{k}} / \partial t = 0$, we get up to the first order in $\Omega\tau$ the following solution for the quasistatic $\boldsymbol{\xi}_{\mathbf{k}}$:

$$\boldsymbol{\xi}_{\mathbf{k}} = \tau (\Omega_{\mathbf{k}} \times \mathbf{s}). \quad (80)$$

The above is a realization of coarse graining, in which we effectively average the spin evolution over the time scales of τ .

Substituting the quasistatic value of $\boldsymbol{\xi}_{\mathbf{k}}$ from Eq. 80 to the time evolution equation for \mathbf{s} , Eq. 76, we find

$$\dot{\mathbf{s}} = \tau \langle \Omega_{\mathbf{k}} \times (\Omega_{\mathbf{k}} \times \mathbf{s}) \rangle. \quad (81)$$

Using the vector product identities we finally obtain for the individual spin components α ,

$$\dot{s}_{\alpha} = \langle \Omega_{\mathbf{k}\alpha} \Omega_{\mathbf{k}\beta} \rangle \tau s_{\beta} - \langle \Omega_{\mathbf{k}}^2 \tau \rangle s_{\alpha}. \quad (82)$$

These equations describe the effective time evolution of the electron spin in the presence of the momentum dependent Larmor precession $\Omega_{\mathbf{k}}$.

For the specific model of the zinc-blend heterostructure with the C_{2v} spin-orbit field, Eq. 73, we obtain the spin dephasing dynamics:

$$\dot{s}_x = -s_x/\tau_s, \quad \dot{s}_y = -s_y/\tau_s, \quad \dot{s}_z = -s_y/\tau_z, \quad (83)$$

where

$$\frac{1}{\tau_{x,y}} = \frac{(\alpha_{BR} \mp \gamma_D)^2}{\alpha_{BR}^2 + \gamma_D^2} \frac{1}{\tau_s}, \quad \frac{1}{\tau_z} = \frac{2}{\tau_s}, \quad (84)$$

and

$$\frac{1}{\tau_s} = \frac{4m}{\hbar^4} \varepsilon_k (\alpha_{BR}^2 + \gamma_D^2). \quad (85)$$

The up (down) sign is for the s_x (s_y). The spin relaxation is anisotropic. The maximum anisotropy is for the case of equal magnitudes of the Bychkov-Rashba and Dresselhaus interactions, $\alpha_{BR} = \pm\gamma_D$. In this case one of the spin components does not decay.⁶ The s_z component of the spin relaxes roughly twice faster than the in-plane components.

0.2.1 The persistent spin helix

Let us consider the case of $\alpha_{BR} = \gamma_D = \lambda/2$. According to Eq. 84 the spin component s_x does not decay, while the decay rates of s_y and s_z are the same:

$$\dot{s}_y = -2s_y/\tau_s, \quad \dot{s}_z = -2s_z/\tau_s. \quad (86)$$

It turns out that a particular nonuniform superposition of s_y and s_z can exhibit no decay as well. This superposition has been termed *persistent spin helix* [13].

Let us assume that the spin is no longer uniform, so that the kinetic equation contains the spin gradient as well, due to the quasiclassical change of the electronic positions:

$$\frac{\partial \mathbf{s}_{\mathbf{k}}}{\partial t} - \boldsymbol{\Omega}_{\mathbf{k}} \times \mathbf{s}_{\mathbf{k}} + \frac{\partial \mathbf{s}_{\mathbf{k}}}{\partial \mathbf{r}} \cdot \mathbf{v}_{\mathbf{k}} = - \sum_{\mathbf{k}'} W_{\mathbf{k}\mathbf{k}'} (\mathbf{s}_{\mathbf{k}} - \mathbf{s}_{\mathbf{k}'}'). \quad (87)$$

A running spin wave,

$$\mathbf{s}_{\mathbf{k}}(\mathbf{r}) = \mathbf{s}_{\mathbf{k}} e^{i\mathbf{q} \cdot \mathbf{r}}; \quad \mathbf{s}_{\mathbf{k}} \equiv \mathbf{s}_{\mathbf{k}}(\mathbf{q}), \quad (88)$$

then evolves according to,

$$\frac{\partial \mathbf{s}_{\mathbf{k}}}{\partial t} - \boldsymbol{\Omega}_{\mathbf{k}} \times \mathbf{s}_{\mathbf{k}} + i(\mathbf{q} \cdot \mathbf{v}_{\mathbf{k}}) \mathbf{s}_{\mathbf{k}} = - \sum_{\mathbf{k}'} W_{\mathbf{k}\mathbf{k}'} (\mathbf{s}_{\mathbf{k}} - \mathbf{s}_{\mathbf{k}'}'). \quad (89)$$

We again separate the fast and slow spins,

$$\mathbf{s}_{\mathbf{k}} = \mathbf{s} + \boldsymbol{\xi}_{\mathbf{k}}, \quad \langle \boldsymbol{\xi}_{\mathbf{k}} \rangle = 0. \quad (90)$$

For the dynamics of the slow part we get,

$$\dot{\mathbf{s}}_{\mathbf{k}} = \langle \boldsymbol{\Omega}_{\mathbf{k}} \times \boldsymbol{\xi}_{\mathbf{k}} \rangle - i \langle (\mathbf{q} \cdot \mathbf{v}_{\mathbf{k}}) \boldsymbol{\xi}_{\mathbf{k}} \rangle. \quad (91)$$

⁶The decay of that component would be due to higher-order (such as cubic) terms in the spin-orbit fields.

Proceeding as in the previous section, assuming that \mathbf{s} is stationary on the time scales relevant to $\boldsymbol{\xi}$, we can write,

$$\dot{\boldsymbol{\xi}}_{\mathbf{k}} - \boldsymbol{\Omega}_{\mathbf{k}} \times \mathbf{s} - \boldsymbol{\Omega}_{\mathbf{k}} \times \boldsymbol{\xi}_{\mathbf{k}} + i(\mathbf{q} \cdot \mathbf{v}_{\mathbf{k}}) \mathbf{s} + i(\mathbf{q} \cdot \mathbf{v}_{\mathbf{k}}) \boldsymbol{\xi}_{\mathbf{k}} = - \sum_{\mathbf{k}'} W_{\mathbf{k}\mathbf{k}'} (\boldsymbol{\xi}_{\mathbf{k}} - \boldsymbol{\xi}_{\mathbf{k}'}). \quad (92)$$

We now work with the following assumptions:

$$\Omega\tau \ll 1, \quad q\ell \ll 1, \quad (93)$$

where $\ell = g\tau$ is the mean free path. We thus assume that the precession is slow on the time scale of the momentum relaxation time, as well as (this is new here) the electronic motion is diffusive on the scale of the wavelength of the spin wave. In the momentum relaxation approximation, also considering the leading terms according to the conditions Eq. 93, we get

$$\dot{\boldsymbol{\xi}}_{\mathbf{k}} - \boldsymbol{\Omega}_{\mathbf{k}} \times \mathbf{s} + i(\mathbf{q} \cdot \mathbf{v}_{\mathbf{k}}) \mathbf{s} = - \frac{\boldsymbol{\xi}}{\tau}. \quad (94)$$

In the steady-state, corresponding to a given $\mathbf{s}(t)$, the solution is

$$\boldsymbol{\xi}_{\mathbf{k}} = \tau (\boldsymbol{\Omega}_{\mathbf{k}} \times \mathbf{s}) - i\tau (\mathbf{q} \cdot \mathbf{v}_{\mathbf{k}}) \mathbf{s}. \quad (95)$$

Substituting to the time evolution equation for \mathbf{s} , Eq. 93, we find

$$\dot{\mathbf{s}} = -2i\tau \langle (\mathbf{q} \cdot \mathbf{v}_{\mathbf{k}}) (\boldsymbol{\Omega}_{\mathbf{k}} \times \mathbf{s}) \rangle + \tau \langle \boldsymbol{\Omega}_{\mathbf{k}} \times (\boldsymbol{\Omega}_{\mathbf{k}} \times \mathbf{s}) \rangle - \tau \langle (\mathbf{q} \cdot \mathbf{v}_{\mathbf{k}})^2 \rangle \mathbf{s}. \quad (96)$$

Using the vector product identities and introducing the diffusivity

$$D = \langle \mathbf{v}_{\mathbf{k}\alpha}^2 \rangle \tau, \quad (97)$$

where α denote the cartesian coordinates (we assume an isotropic system), we finally obtain

$$\dot{s}_{\alpha} = -2i\tau \varepsilon_{\alpha\beta\gamma} q_{\delta} \langle v_{\mathbf{k}\delta} \Omega_{\mathbf{k}\beta} \rangle s_{\gamma} + \langle \Omega_{\mathbf{k}\alpha} \Omega_{\mathbf{k}\beta} \rangle \tau s_{\beta} - \langle \Omega_{\mathbf{k}}^2 \tau \rangle s_{\alpha} - Dq^2 s_{\alpha}. \quad (98)$$

For our specific case of

$$\boldsymbol{\Omega}_{\mathbf{k}} = (\lambda k_y, 0, 0), \quad \lambda = \alpha_{BR} + \beta_D, \quad (99)$$

we find that s_x decays only via diffusion:

$$\dot{s}_x = -Dq^2 s_x. \quad (100)$$

The spin dephasing is ineffective. This is an expected result.

More interesting behavior is found for the two remaining spin components, s_y and s_z . These two spin components, transverse to the spin-orbit field, are coupled:

$$\dot{s}_y = +2i \frac{m\lambda}{\hbar} q_y D s_z - (\tau \Omega^2 + Dq^2) s_y, \quad (101)$$

$$\dot{s}_z = -2i \frac{m\lambda}{\hbar} q_y D s_y - (\tau \Omega^2 + Dq^2) s_z, \quad (102)$$

and we denoted $\Omega^2 \equiv \langle \Omega_{\mathbf{k}}^2 \rangle$. Interestingly, this set of coupled differential equations has only decaying solutions. This is best seen by looking at the rotating spins:

$$s_+ = s_y + i s_z, \quad s_- = s_y - i s_z, \quad (103)$$

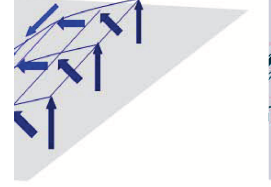


Figure 7: The persistent spin helix is a wave of circularly polarized spin. The sense of polarization, clock or counterclockwise, depends on the relative sign of the Dresselhaus and the Bychkov-Rashba spin-orbit coupling.

whose time evolutions are uncoupled,

$$\dot{s}_+ = - \left(\Omega^2 \tau + Dq^2 - \frac{2m\lambda}{\hbar} q_y D \right) s_+, \quad (104)$$

$$\dot{s}_- = - \left(\Omega^2 \tau + Dq^2 + \frac{2m\lambda}{\hbar} q_y D \right) s_- \quad (105)$$

Considering that

$$\Omega^2 \tau = \lambda^2 \langle k_y^2 \rangle \tau = \lambda^2 \left(\frac{m}{\hbar} \right)^2 \langle v_y^2 \rangle \tau = \lambda^2 \left(\frac{m}{\hbar} \right)^2 D, \quad (106)$$

we find that the decay of s_+ vanishes for the wave vector

$$q_y^{\text{PSH}} = \frac{m\lambda}{\hbar} = \frac{m}{\hbar} (\alpha_{BR} + \beta_D). \quad (107)$$

The abbreviation PSH stands for the *persistent spin helix*, which is the spin wave described by s_+ at this particular wave vector. See Fig. 7. While individually both s_y and s_z decay at a generic wave vector (and also in the uniform case, $q = 0$), the spin helix they form does not decay in the approximation of the linear spin-orbit field. The spin wave rotating in the opposite sense, s_- , on the other hand, decays. And vice versa for $q_y^{\text{PSH}} = -m\lambda/\hbar$. The persistent spin helix was observed in the spin grating experiment [14].

0.3 The Elliott-Yafet mechanism

Elliott [1] was first to recognize the role of the intrinsic spin-orbit coupling—that coming from the host ions—on spin relaxation. Yafet [2] significantly extended this theory to properly treat electron-phonon spin-flip scattering. The Elliott-Yafet mechanism dominates the spin relaxation of conduction electrons in elemental metals and semiconductors, the systems with space inversion symmetry. In systems lacking this symmetry, the mechanism appears competes with the D'yakonov-Perel' one; the dominance of one over the other depends on the material in question and specific conditions, such as temperature and doping.

Suppose a nonequilibrium spin accumulates in a nonmagnetic degenerate conductor. The spin accumulation is given as the difference between the chemical potentials for the spin up and the spin down electrons. Let us denote the corresponding potentials by μ_\uparrow and μ_\downarrow . The nonequilibrium electron occupation function for the spin λ is

$$f_{\lambda\mathbf{k}} \approx \frac{1}{e^{\beta(\varepsilon_{\mathbf{k}} - \mu_\lambda)} + 1} \approx f_{\mathbf{k}}^0 + \left[-\frac{\partial f_{\mathbf{k}}^0}{\partial \varepsilon_{\mathbf{k}}} \right] (\mu_\lambda - \varepsilon_F). \quad (108)$$

Here,

$$f_{\mathbf{k}}^0 = f^0(\varepsilon_{\mathbf{k}}) = \frac{1}{e^{\beta(\varepsilon_{\mathbf{k}} - \varepsilon_F)} + 1}, \quad (109)$$

describes the equilibrium degenerate electronic system of the Fermi energy ε_F . The electron density for the spin λ is

$$n_\lambda = \int d\varepsilon g_s(\varepsilon) f_\lambda^0(\varepsilon) \approx \frac{n}{2} + \int d\varepsilon g(\varepsilon) \left[-\frac{\partial f^0(\varepsilon)}{\partial \varepsilon} \right] (\mu_\lambda - \varepsilon_F). \quad (110)$$

Here $g_s(\varepsilon)$ is the electronic density of states, defined per unit volume and per spin, at the fermi level:

$$g_s = \sum_{\mathbf{k}} \left[-\frac{\partial f_{\mathbf{k}}^0}{\partial \varepsilon_{\mathbf{k}}} \right]. \quad (111)$$

The total electron density n is

$$n = 2 \int d\varepsilon g(\varepsilon) f^0(\varepsilon). \quad (112)$$

We assume that the spin accumulation does not charge the system, that is, the charge neutrality is preserved $n_\uparrow + n_\downarrow = n$. This condition is well satisfied in metals and degenerate semiconductors that we consider. We then get

$$\mu_\uparrow + \mu_\downarrow = 2\varepsilon_F. \quad (113)$$

The spin density is,

$$s = n_\uparrow - n_\downarrow = g_s(\mu_\uparrow - \mu_\downarrow) = g_s \mu_s, \quad (114)$$

where μ_s is the spin quasichemical potential, $\mu_s = (\mu_\uparrow - \mu_\downarrow)$.

The spin relaxation time T_1 is defined by the decay law,

$$\frac{ds}{dt} = \frac{dn_\uparrow}{dt} - \frac{dn_\downarrow}{dt} = W_{\uparrow\downarrow} - W_{\downarrow\uparrow} = -\frac{s}{T_1} = -g_s \frac{\mu_s}{T_1}. \quad (115)$$

Here $W_{\uparrow\downarrow}$ is the net number of transitions per unit time from the spin \downarrow to \uparrow . Similarly, $W_{\downarrow\uparrow}$ expresses the rate of spin flips from \uparrow to \downarrow . In the degenerate conductors the spin decay is directly proportional to the decay of the spin accumulation μ_s :

$$\frac{d\mu_s}{dt} = -\frac{\mu_s}{T_1}. \quad (116)$$

0.3.1 The electron-impurity scattering

We need to distinguish the cases of the impurity or host induced spin-orbit coupling. Although the formulas for the calculation of T_1 look similar in the two cases, they are nevertheless conceptually different.

Spin-orbit coupling by the impurity

If the spin-orbit coupling comes from the impurity potential (in this case the coupling is often termed *extrinsic*) the spin-flip scattering is due to that potential.

The number of transitions per unit time from the spin up to the spin down states is

$$W_{\uparrow\downarrow} = \sum_{\mathbf{k}n} \sum_{\mathbf{k}'n'} W_{\mathbf{k}'n'\uparrow, \mathbf{k}n\downarrow} - W_{\mathbf{k}n\downarrow, \mathbf{k}'n'\uparrow}. \quad (117)$$

The rate is given by the spin-flip events from down to up minus the ones from up to down. We use the Fermi golden rule to write out the individual scattering rates:

$$W_{\mathbf{k}'n'\uparrow, \mathbf{k}n\downarrow} = \frac{2\pi}{\hbar} f_{\mathbf{k}n} (1 - f'_{\mathbf{k}'n}) |U_{\mathbf{k}n\uparrow, \mathbf{k}'n'\downarrow}|^2 \delta(\varepsilon_{\mathbf{k}'n'} - \varepsilon_{\mathbf{k}n}), \quad (118)$$

$$W_{\mathbf{k}n\uparrow, \mathbf{k}'n'\downarrow} = \frac{2\pi}{\hbar} f_{\mathbf{k}'n'} (1 - f_{\mathbf{k}n}) |U_{\mathbf{k}'n'\downarrow, \mathbf{k}n\uparrow}|^2 \delta(\varepsilon_{\mathbf{k}'n'} - \varepsilon_{\mathbf{k}n}). \quad (119)$$

Substituting for the occupation numbers the linearized expression, Eq. 108, and using the definition of the spin relaxation of Eq. 115, we obtain for the spin relaxation rate the expression,

$$\frac{1}{T_1} = \frac{2\pi}{\hbar} \frac{1}{g_s} \sum_{\mathbf{k}\mathbf{k}'} |U_{\mathbf{k}n\uparrow, \mathbf{k}'n'\downarrow}|^2 \left[-\frac{\partial f^0(\varepsilon_{\mathbf{k}})}{\partial \varepsilon_{\mathbf{k}}} \right] \delta(\varepsilon_{\mathbf{k}'n'} - \varepsilon_{\mathbf{k}n}). \quad (120)$$

If we define the spin relaxation for the individual momentum state \mathbf{k} as

$$\frac{1}{T_{1\mathbf{k}}} = \frac{2\pi}{\hbar} \sum_{\mathbf{k}'} |U_{\mathbf{k}n\uparrow, \mathbf{k}'n'\downarrow}|^2 \left[-\frac{\partial f^0(\varepsilon_{\mathbf{k}})}{\partial \varepsilon_{\mathbf{k}}} \right] \delta(\varepsilon_{\mathbf{k}'n'} - \varepsilon_{\mathbf{k}n}), \quad (121)$$

which has a straightforward interpretation as the spin-flip rate by the elastic impurity scattering to all the possible states \mathbf{k}' , we get

$$\frac{1}{T_1} = \left\langle \frac{1}{T_{1\mathbf{k}}} \right\rangle_{\varepsilon_{\mathbf{k}} = \varepsilon_F}, \quad (122)$$

as the average of the individual scattering rates over the electronic Fermi surface.

Spin-orbit coupling by the host lattice

If the spin-orbit coupling comes from the host lattice (in this case the coupling is often termed *intrinsic*) the spin-flip scattering is due to the admixture of the Pauli spin up and spin down states in the Bloch eigenstates.

How do the Bloch states actually look like in the presence of spin-orbit coupling? Elliott showed that the Bloch states corresponding to a generic lattice wave vector \mathbf{k} and band n can be written as [1],

$$\Psi_{\mathbf{k}, n\uparrow}(\mathbf{r}) = [a_{\mathbf{k}n}(\mathbf{r})|\uparrow\rangle + b_{\mathbf{k}n}(\mathbf{r})|\downarrow\rangle] e^{i\mathbf{k}\cdot\mathbf{r}}, \quad (123)$$

$$\Psi_{\mathbf{k}, n\downarrow}(\mathbf{r}) = [a_{-\mathbf{k}n}^*(\mathbf{r})|\downarrow\rangle - b_{-\mathbf{k}n}^*(\mathbf{r})|\uparrow\rangle] e^{i\mathbf{k}\cdot\mathbf{r}}. \quad (124)$$

The states $|\uparrow\rangle$ and $|\downarrow\rangle$ are the usual Pauli spinors. We can select the two states such that $|a_{\mathbf{k}n}| \approx 1$ while $|b_{\mathbf{k}n}| \ll 1$, due to the weak spin orbit coupling; this justifies calling the two above states “spin up” (\uparrow) and “spin down” (\downarrow). In fact, to “prepare” the states for the calculation of the spin relaxation, they need to satisfy,

$$\langle \mathbf{k}, n\lambda | \sigma_z | \mathbf{k}, n\lambda' \rangle = \lambda \delta_{\lambda\lambda'}, \quad (125)$$

with $\lambda = \uparrow, \downarrow$. That is, the two states should diagonalize the spin matrix S_z (or whatever spin direction one is interested in).

The Bloch states of Eqs. 123 and 124 allow for a spin flip even if the impurity does not induce a spin-orbit coupling. Indeed, the matrix element

$$\langle \mathbf{k}, n \uparrow | U | \mathbf{k}, n \downarrow \rangle \sim ab, \quad (126)$$

is in general non zero due to the spin admixture. The spin flip probability is proportional to $|b|^2$, the spin admixture probability. This quantity is crucial in estimating the spin relaxation in the Elliott-Yafet mechanism. The spin relaxation time T_1 in this case can be calculated using the formula Eq. 120, with

$$U_{\mathbf{k}n\uparrow, \mathbf{k}'n'\downarrow} = U_{\mathbf{k}n\downarrow, \mathbf{k}'n'\uparrow}, \quad (127)$$

given by Eq. 126. A useful *rule of thumb* for estimating the spin relaxation time in this case is

$$\frac{1}{T_1} \approx \frac{\langle b_{\mathbf{k}n}^2 \rangle}{\tau_p}, \quad (128)$$

where the averaging of the spin admixture probabilities b^2 is performed over the Fermi surface (or the relevant energy scales of the problem); τ_p is the spin-conserving momentum relaxation time. We stress that b is obtained from the states prepared according to Eq. 125.

0.3.2 The electron-phonon scattering

The spin-flip due to the scattering of the electrons off of phonons involves the intrinsic spin-orbit potential. The electron Bloch states are the ones given by the Eqs. 123 and 124, prepared according to Eq. 125.

The net number of transitions per unit time from the spin up to the spin down states is

$$W_{\uparrow\downarrow} = \sum_{\mathbf{k}n} \sum_{\mathbf{k}'n'} \sum_{\mathbf{q}\nu} W_{\mathbf{k}n\uparrow, \mathbf{q}\nu; \mathbf{k}'n'\downarrow} + W_{\mathbf{k}n\downarrow, \mathbf{q}\nu; \mathbf{k}'n'\uparrow} - W_{\mathbf{k}'n'\downarrow, \mathbf{q}\nu; \mathbf{k}n\uparrow} - W_{\mathbf{k}'n'\uparrow, \mathbf{q}\nu; \mathbf{k}n\downarrow}. \quad (129)$$

We introduced the rates of the spin flip transitions accompanied by the phonon absorption and emissions as follows. The net transition rate from the single electron state $|\mathbf{k}'n'\downarrow\rangle$ to the electron state $|\mathbf{k}n\uparrow\rangle$ while the phonon of momentum \mathbf{q} and polarization ν is emitted, is

$$W_{\mathbf{k}n\uparrow, \mathbf{q}\nu; \mathbf{k}'n'\downarrow} = \frac{2\pi}{\hbar} |M_{\mathbf{k}n\uparrow, \mathbf{q}\nu; \mathbf{k}'n'\downarrow}|^2 f_{\mathbf{k}'n'\downarrow} (1 - f_{\mathbf{k}n\uparrow}) \delta(\varepsilon_{\mathbf{k}n} - \varepsilon_{\mathbf{k}'n'} + \hbar\omega_{\mathbf{q}\nu}). \quad (130)$$

Similarly, the net transition rate from the single electron state $|\mathbf{k}'n'\downarrow\rangle$ to the electron state $|\mathbf{k}n\uparrow\rangle$ while the phonon of momentum \mathbf{q} and polarization ν is absorbed, is

$$W_{\mathbf{k}n\downarrow, \mathbf{q}\nu; \mathbf{k}'n'\uparrow} = \frac{2\pi}{\hbar} |M_{\mathbf{k}n\downarrow, \mathbf{q}\nu; \mathbf{k}'n'\uparrow}|^2 f_{\mathbf{k}'n'\uparrow} (1 - f_{\mathbf{k}n\downarrow}) \delta(\varepsilon_{\mathbf{k}n} - \varepsilon_{\mathbf{k}'n'} - \hbar\omega_{\mathbf{q}\nu}). \quad (131)$$

The same way are defined the remaining two rates, $W_{\mathbf{k}'n'\downarrow;\mathbf{q}\nu;\mathbf{k}n\uparrow}$ for the spin flip from $\mathbf{k}n \uparrow$ to $\mathbf{k}'n' \downarrow$ with the phonon emission, and $W_{\mathbf{k}'n'\downarrow;\mathbf{k}n\uparrow\mathbf{q}\nu}$ for the phonon absorption:

$$W_{\mathbf{k}'n'\downarrow;\mathbf{q}\nu;\mathbf{k}n\uparrow} = \frac{2\pi}{\hbar} |M_{\mathbf{k}'n'\downarrow;\mathbf{q}\nu;\mathbf{k}n\uparrow}|^2 f_{\mathbf{k}n\uparrow} (1 - f_{\mathbf{k}'n'\downarrow}) \delta(\varepsilon_{\mathbf{k}'n'} - \varepsilon_{\mathbf{k}n} + \hbar\omega_{\mathbf{q}\nu}), \quad (132)$$

$$W_{\mathbf{k}'n'\downarrow;\mathbf{k}n\uparrow\mathbf{q}\nu} = \frac{2\pi}{\hbar} |M_{\mathbf{k}'n'\downarrow;\mathbf{q}\nu;\mathbf{k}n\uparrow}|^2 f_{\mathbf{k}n\uparrow} (1 - f_{\mathbf{k}'n'\downarrow}) \delta(\varepsilon_{\mathbf{k}'n'} - \varepsilon_{\mathbf{k}n} - \hbar\omega_{\mathbf{q}\nu}). \quad (133)$$

The calculation of the spin relaxation due to the electron-phonon scattering is rather involved and we cite here only the final result for degenerate conductors:

$$\begin{aligned} \frac{1}{T_1} = & \frac{4\pi}{\hbar} \frac{1}{g_s} \sum_{\mathbf{k}n} \sum_{\mathbf{k}'n'} \sum_{\nu} \left(-\frac{\partial f_{\mathbf{k}n}^0}{\varepsilon_{\mathbf{k}n}} \right) \frac{\hbar}{2NM} \frac{N^2}{\omega_{\mathbf{k}'-\mathbf{k},\nu}} |\epsilon_{\mathbf{k}-\mathbf{k}',\nu} \cdot \langle \mathbf{k}n \uparrow | \nabla V | \mathbf{k}'n' \downarrow \rangle|^2 \\ & \times \{ [n_{\mathbf{q}\nu} - f_{\mathbf{k}'n'}^0 + 1] \delta(\varepsilon_{\mathbf{k}n} - \varepsilon_{\mathbf{k}'n'} - \hbar\omega_{\mathbf{q}\nu}) + [n_{\mathbf{q}\nu} + f_{\mathbf{k}'n'}^0] \delta(\varepsilon_{\mathbf{k}n} - \varepsilon_{\mathbf{k}'n'} + \hbar\omega_{\mathbf{q}\nu}) \} \end{aligned} \quad (134)$$

The electronic bands are described by the energies $\varepsilon_{\mathbf{k}n}$ of the state with momentum \mathbf{k} and band index n . Phonon frequencies are $\omega_{\mathbf{q}\nu}$, for the phonon of momentum \mathbf{q} and polarization ν ; similarly for the phonon polarization vector $\epsilon_{\mathbf{q}\nu}$. We further denoted by M the atomic mass, by N the number of atoms in the lattice, and by ∇V the gradient of the electron-lattice ion potential. The equilibrium phonon occupation numbers are denoted as $n_{\mathbf{q}\nu}$, given as

$$n_{\mathbf{q}\nu} = n(\omega_{\mathbf{q}\nu}) = \frac{1}{e^{\beta\hbar\omega_{\mathbf{q}\nu}} - 1}. \quad (136)$$

The electronic states $|\mathbf{k}n\sigma\rangle$ are normalized to the whole space.

Two types of processes contribute to the phonon induced spin flips. First, what we call the *Elliott processes*, are the Elliott-type of spin flips in which the Bloch states given by Eq. 123 and 124 scatter by the scalar part of the gradient of the electron-ion potential V . Second, what we call the *Yafet processes*, are the spin flips due to the gradient of the spin-orbit part of the electron-ion potential. These two processes are typically of similar order of magnitude and have to be added coherently in order to obtain T_1 .

Figure 8 shows the experimental data of the spin relaxation in intrinsic (nondegenerate) silicon, obtained by the spin resonance [16, 8] and the spin injection [17] experiments. The calculation based on the Elliott-Yafet mechanism of the phonon induced spin flips reproduces the experiments [15].

The Yafet relation

The expected temperature dependence of the phonon-induced spin flips in degenerate conductors, is, according to the Elliott-Yafet mechanism, $1/T_1 \sim T$ at high temperatures (starting roughly at a fraction of the Debye temperature T_D) and $1/T_1 \sim T^3$ at low temperatures, in analogy with the conventional spin-conserving electron-phonon scattering. The high-temperature dependence originates from the linear increase of the phonon occupation numbers n with increasing temperature: $n_{\mathbf{q}\nu} \sim T$, as $k_B T \gg \omega_{\mathbf{q}\nu}$. At $T > T_D$ are all the phonons excited. The low temperature dependence of the spin-conserving scattering follows from setting the relevant phonon energy scale to $k_B T$. The matrix element

$$\langle \mathbf{k}n \uparrow | \nabla V | \mathbf{k}'n' \uparrow \rangle \sim q, \quad (137)$$

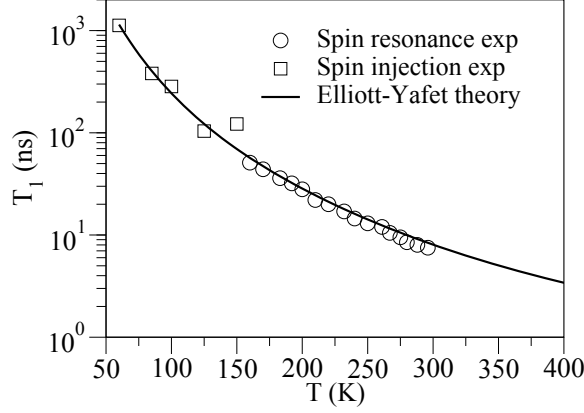


Figure 8: *Spin relaxation in silicon.* Phonon-induced spin relaxation in silicon results in an approximate T^3 power law [15]. Shown are the experimental data from spin resonance [16] and spin injection [17] experiments, and a calculation based on the Elliott-Yafet mechanism [15].

which gives the $1/T_1 \sim T^3$ dependence. However, Yafet showed [2] that for the spin-flip matrix element the space inversion symmetry modifies the momentum dependence to

$$\langle \mathbf{k}n \uparrow | \nabla V | \mathbf{k}'n' \downarrow \rangle \sim q^2, \quad (138)$$

so that

$$\frac{1}{T_1} \sim T^5, \quad (139)$$

instead of the expected T^3 . Since the same temperature dependence holds for the phonon-induced electrical resistance $\rho(T)$, we can write

$$\frac{1}{T_1} \sim \rho(T), \quad (140)$$

known as the *Yafet relation*.

The relation Eq. 138 holds if both the Elliott and Yafet processes are taken into account. Individually, they would lead to a linear dependence on q . The quantum mechanical interference between these two processes thus significantly reduces the spin-flip probability at low momenta q . An example is shown in Fig. 9. The Elliott and Yafet processes would individually give much stronger spin relaxation than is observed. Their destructive interference can modify T_1 by orders of magnitude.

0.4 Results based on kinetic spin Bloch equation approach

It was shown by Wu *et al.* from a full microscopic kinetic-spin-Bloch-equation approach [9] that the single-particle approach is inadequate in accounting for the spin relaxation/dephasing

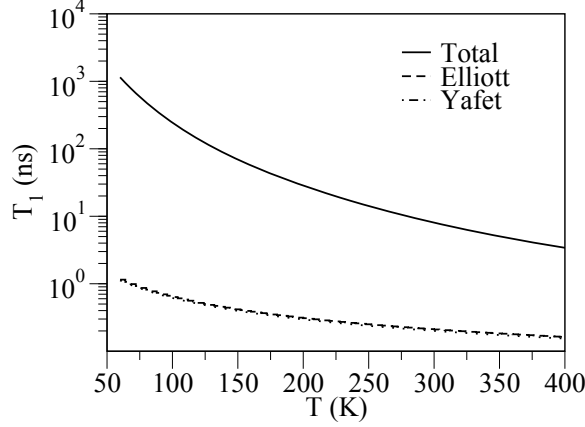


Figure 9: *Interference between the Elliott and Yafet processes.* Individually the Elliott and Yafet phonon-induced spin relaxation processes give spin relaxation orders of magnitude stronger than the total one. This example is from the calculation of T_1 in silicon [15].

both in the time [18, 19, 20, 21, 22] and in the space [23, 24, 25] domains. The momentum dependence of the effective magnetic field (the D'yakonov-Perel' term) and the momentum dependence of the spin diffusion rate along the spacial gradient [23] or even the random spin-orbit coupling [26] all serve as inhomogeneous broadenings [19, 20]. It was pointed out that in the presence of inhomogeneous broadening, any scattering, including the carrier-carrier Coulomb scattering, can cause an irreversible spin relaxation/dephasing [19]. Moreover, besides the spin relaxation/dephasing channel the scattering provides, it also gives rise to the counter effect to the inhomogeneous broadening. The scattering tends to drive carriers to a more homogeneous state and therefore suppresses the inhomogeneous broadening. Finally, this approach is valid in both strong and weak scattering limits and also can be used to study systems far away from the equilibrium, thanks to the inclusion of the Coulomb scattering.

In the following, we present the main results based on kinetic-spin-Bloch-equation approach. We first briefly introduce the kinetic spin Bloch equations. Then we review the results of the spin relaxation/dephasing in the time and space domains respectively.

0.4.1 Kinetic spin Bloch equations

By using the nonequilibrium Green function method with gradient expression as well as the generalized Kadanoff-Baym Ansatz [27], we construct the kinetic spin Bloch equations as follows:

$$\begin{aligned} \dot{\rho}_{\mathbf{k}}(\mathbf{r}, t) = & \dot{\rho}_{\mathbf{k}}(\mathbf{r}, t)|_{\text{dr}} + \dot{\rho}_{\mathbf{k}}(\mathbf{r}, t)|_{\text{dif}} \\ & + \dot{\rho}_{\mathbf{k}}(\mathbf{r}, t)|_{\text{coh}} + \dot{\rho}_{\mathbf{k}}(\mathbf{r}, t)|_{\text{scat}} . \end{aligned} \quad (141)$$

Here $\rho_{\mathbf{k}}(\mathbf{r}, t)$ are the density matrices of electrons with momentum \mathbf{k} at position \mathbf{r} and time t . The off-diagonal elements of $\rho_{\mathbf{k}}$ represent the correlations between the conduction and valence bands, different subbands (in confined structures) and different spin states. $\dot{\rho}_{\mathbf{k}}(\mathbf{r}, t)|_{\text{dr}}$ are the driving terms from the external electric field. The coherent terms in Eq.

(141) $\dot{\rho}_{\mathbf{k}}|_{\text{coh}}$ are composed of the energy spectrum, magnetic field and effective magnetic field from the D'yakonov-Perel' term, and the Coulomb Hartree-Fock terms. The diffusion terms $\dot{\rho}_{\mathbf{k}}(\mathbf{r}, t)|_{\text{dif}}$ come from the spacial gradient. The scattering terms $\dot{\rho}_{\mathbf{k}}(\mathbf{r}, t)|_{\text{scat}}$ include the spin-flip and spin conserving electron-electron, electron-phonon and electron-impurity scatterings. The spin-flip terms correspond to the Elliot-Yafet and/or Bir-Aronov-Pikus mechanisms. Detailed expressions of these terms in the kinetic spin Bloch equations depend on the band structures, doping situations and dimensionalities [9] and can be found in the literature for different cases, such as intrinsic quantum wells [18], n -type quantum wells without [21, 28, 29, 30, 31] and with [22, 32, 33, 34] electric field, p -type quantum wells [35, 36, 37], quantum wires [38, 39], quantum dots [40] and bulk materials [41] in the spacial uniform case and quantum wells in spacial non-uniform case [23, 24, 42, 43]. By numerically solving the kinetic spin Bloch equations with all the scattering explicitly included, one is able to obtain the time evolution and/or spacial distribution of the density matrices, and hence all the measurable quantities, such as mobility, diffusion constant, optical relaxation/dephasing time, spin relaxation/dephasing time, spin diffusion length, as well as hot-electron temperature, can be determined from the theory without any fitting parameters.

0.4.2 Spin relaxation/dephasing

In this subsection we present the main understandings of the spin relaxation/dephasing added to the literature from the kinetic-spin-Bloch-equation approach. We focus on three related issues: (i) The importance of the Coulomb interaction to the spin relaxation/dephasing; (ii) Spin dynamics far away from the equilibrium; and (iii) Qualitatively different behaviors from those wildly adopted in the literature.

First we address the effect of the Coulomb interaction. Based on the single-particle approach, it has been long believed that the Coulomb scattering is irrelevant to the spin relaxation/dephasing [44]. It was first pointed out by Wu and Ning [19] that in the presence of inhomogeneous broadening in spin precession, i.e., the spin precession frequencies are \mathbf{k} -dependent, any scattering, including the spin-conserving scattering, can cause irreversible spin dephasing. This inhomogeneous broadening can come from the energy-dependent g -factor [19], the D'yakonov-Perel' term [20], the random spin-orbit coupling [26], and even the momentum dependence of the spin diffusion rate along the spacial gradient [23]. Wu and Ning first showed that with the energy-dependent g -factor as an inhomogeneous broadening, the Coulomb scattering can lead to irreversible spin dephasing [19]. In [001]-grown n -doped quantum wells, the importance of the Coulomb scattering for spin relaxation/dephasing was proved by Glazov and Ivchenko [45] by using perturbation theory and by Weng and Wu [21] from the kinetic-spin-Bloch-equation approach. In a temperature-dependent study of the spin dephasing in [001]-oriented n -doped quantum wells, Leyland et al. experimentally verified the effects of the electron-electron Coulomb scattering by closely measuring the momentum scattering rate from the mobility [46]. By showing the momentum relaxation rate obtained from the mobility cannot give the correct spin relaxation rate, they showed the difference comes from the Coulomb scattering. Later Zhou et al. even predicted a peak from the Coulomb scattering in the temperature dependence of the spin relaxation time in a high-mobility low-density n -doped (001) quantum well [29]. This was later demonstrated by Ruan et al. experimentally [47].

Figure 10 shows the temperature dependence of the spin relaxation time of a 7.5 nm GaAs/Al_{0.4}Ga_{0.6}As quantum well at different electron and impurity densities [29]. For this small well width, only the lowest subband is needed in the calculation. It is shown in the

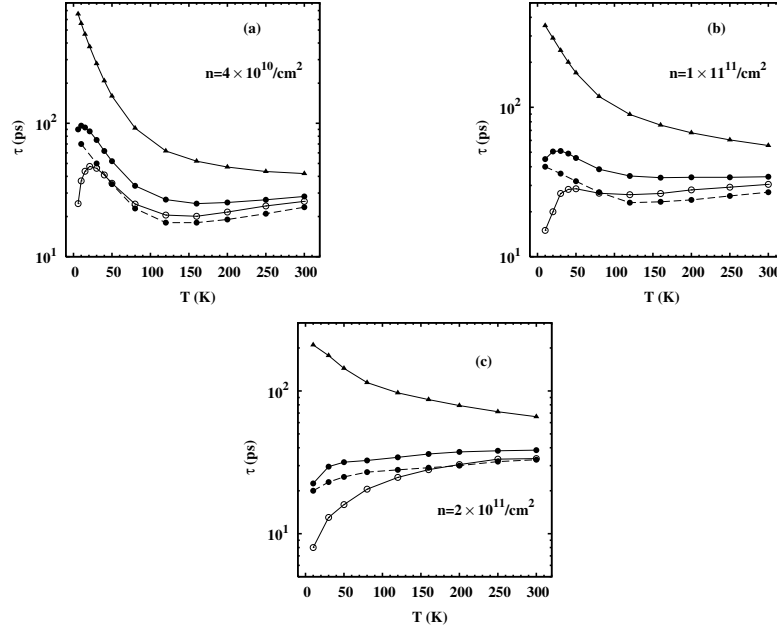


Figure 10: Spin relaxation time τ vs. the temperature T with well width $a = 7.5$ nm and electron density n being (a) $4 \times 10^{10} \text{ cm}^{-2}$, (b) $1 \times 10^{11} \text{ cm}^{-2}$, and (c) $2 \times 10^{11} \text{ cm}^{-2}$, respectively. Solid curves with triangles: impurity density $n_i = n$; solid curves with dots: $n_i = 0.1n$; solid curves with circles: $n_i = 0$; dashed curves with dots: $n_i = 0.1n$ and no Coulomb scattering. From Zhou et al. [29].

figure that when the electron-impurity scattering is dominant, the spin relaxation time decreases with increasing temperature monotonically. This is in good agreement with the experimental findings [48] and a nice agreement of the theory and the experimental data from 20 to 300 K is given in Ref. [29]. However, it is shown that for sample with high mobility, i.e., low impurity density, when the electron density is low enough, there is a peak at low temperature. This peak, located around the Fermi temperature of electrons $T_F^e = E_F/k_B$, is identified to be solely due to the Coulomb scattering [29, 49]. It disappears when the Coulomb scattering is switched off, as shown by the dashed curves in the figure. This peak also disappears at high impurity densities. It is also noted in Fig. 10(c) that for electrons of high density so that T_F^e is high enough and the contribution from the electron-longitudinal optical-phonon scattering becomes marked, the peak disappears even for sample with no impurity and the spin relaxation time increases with temperature monotonically. The physics leading to the peak is due to the crossover of the Coulomb scattering from the degenerate to the non-degenerate limit. At $T < T_F^e$, electrons are in the degenerate limit and the electron-electron scattering rate $1/\tau_{ee} \propto T^2$. At $T > T_F^e$, $1/\tau_{ee} \propto T^{-1}$ [45, 50]. Therefore, at low electron density so that T_F^e is low enough and the electron-acoustic phonon scattering is very weak comparing with the electron-electron Coulomb scattering, the Coulomb scattering is dominant for high mobility sample. Hence the different temperature dependence of the Coulomb scattering leads to the peak. It is noted that the peak is just a feature of the crossover from the degenerate to the non-degenerate limit. The location of the peak also depends on the strength of the inhomogeneous broadening. When the inhomogeneous broadening depends on momentum linearly, the peak tends to appear at the Fermi temperature. A similar peak was predicted in the electron spin relaxation in *p*-type GaAs quantum well

and the hole spin relaxation in (001) strained asymmetric Si/SiGe quantum well, where the electron and hole spin relaxation times both show a peak at the hole Fermi temperature T_F^h [37, 36]. When the inhomogeneous broadening depends on momentum cubically, the peak tends to shift to a lower temperature. It was predicted that a peak in the temperature dependence of the electron spin relaxation time appears at a temperature in the range of $(T_F^e/4, T_F^e/2)$ in the intrinsic bulk GaAs [41] and a peak in the temperature dependence of the hole spin relaxation time at $T_F^h/2$ in p -type Ge/SiGe quantum well [37]. Ruan *et al.* demonstrated the peak experimentally in a high-mobility low-density GaAs/Al_{0.35}Ga_{0.65}As heterostructure and showed a peak appears at $T_F^e/2$ in the spin relaxation time versus temperature curve [47].

For larger well width, the situation may become different in the non-degenerate limit. Weng and Wu calculated the spin relaxation/dephasing for (001) GaAs quantum wells with larger well width and high mobility, by including the multi-subband effect [28]. It is shown that for small/large well width so that the linear/cubic Dresselhaus term is dominant, the spin relaxation/dephasing time increases/decreases with the temperature. This is because with the increase of temperature, both the inhomogeneous broadening and the scattering get enhanced. The relative importance of these two competing effects is different when the linear/cubic term is dominant [28]. Jiang and Wu further introduced strain to change the relative importance of the linear and cubic D'yakonov-Perel' terms and showed the different temperature dependences of the spin relaxation time [51]. This prediction has been realized experimentally by Holleitner *et al.* where they showed that in n -type two-dimensional InGaAs channels, when the linear D'yakonov-Perel' term is suppressed, the spin relaxation time decreases with temperature monotonically [52]. Another interesting prediction related to the multi-subband effect is related to the effect of the inter-subband Coulomb scattering. From the calculation Weng and Wu found out that although the inhomogeneous broadening from the higher subband of the (001) quantum well is much larger, due to the strong inter-subband Coulomb scattering, the spin relaxation times of the lowest two subbands are identical [28]. This prediction has later been verified experimentally by Zhang *et al.*, who studied the spin dynamics in a single-barrier heterostructure by time-resolved Kerr rotation [53]. By applying a gate voltage, they effectively manipulated the confinement of the second subband and the measured spin relaxation times of the first and second subbands are almost identical at large gate voltage. Lü *et al.* showed that due to the Coulomb scattering, $T_2 = T_2^*$ in (001) GaAs quantum wells for a wide temperature and density regime [54]. It was also pointed out by Lü *et al.* that in the strong (weak) scattering limit, introducing the Coulomb scattering will always lead to a faster (slower) spin relaxation/dephasing [35].

Another important effect from the Coulomb interaction to the spin relaxation/dephasing comes from the Coulomb Hartree-Fock contribution in the coherent terms of the kinetic spin Bloch equations. Weng and Wu [21] first pointed out that at a high spin polarization, the Hartree-Fock term serves as an effective magnetic field along the z axis which blocks the spin precession. As a result, the spin relaxation/dephasing time increases dramatically with the spin polarization. They further pointed out that the spin relaxation/dephasing time decreases with temperature at high spin polarization in quantum well with small well width, which is in contrast to the situation with small spin polarizations. These predictions have been verified experimentally by Stich *et al.* in an n -type (001) GaAs quantum well with high mobility [55, 56]. By changing the intensity of the circularly polarized lasers, Stich *et al.* measured the spin dephasing time in a high mobility n -type GaAs quantum well as a function of initial spin polarization. Indeed they observed an increase of the spin dephasing time with the increased spin polarization, and the theoretical calculation based on the kinetic spin Bloch equations nicely reproduced the experimental findings when the Hartree-Fock term

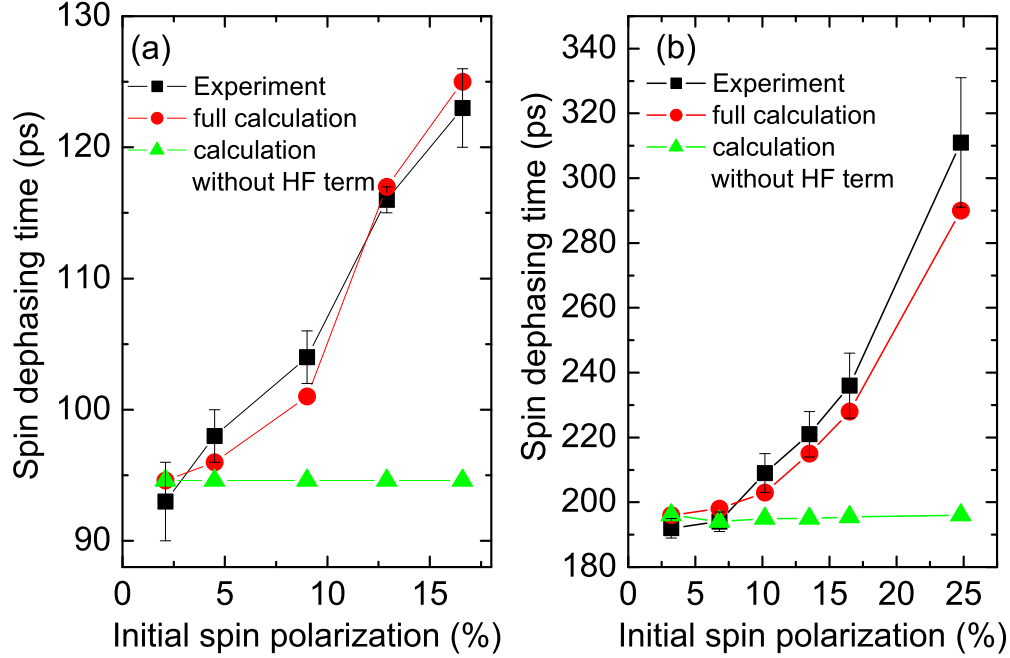


Figure 11: (a) The spin dephasing times as a function of initial spin polarization for constant, *low* excitation density and variable polarization degree of the pump beam. The measured spin dephasing times are compared to calculations with and without the Hartree-Fock (HF) term, showing its importance. (b) The spin dephasing times measured and calculated for constant, *high* excitation density and variable polarization degree. From Stich *et al.* [56].

was included [55]. It was also shown that when the Hartree-Fock term is removed, one does not see any increase of the spin dephasing time. Later, they further improved the experiment by replacing the circular-polarized laser pumping with the elliptic polarized laser pumping. By doing so, they were able to vary the spin polarization without changing the carrier density. Figure. 11 shows the measured spin dephasing times as function of initial spin polarization under two fixed pumping intensities, together with the theoretical calculations with and without the Coulomb Hartree-Fock term. Again the spin dephasing time increases with the initial spin polarization as predicted and the theoretical calculations with the Hartree-Fock term are in good agreement with the experimental data [56]. Moreover, Stich *et al.* also confirmed the prediction of the temperature dependences of the spin dephasing time at low and high spin polarizations [56]. Figure 12(a) shows the measured temperature dependences of the spin dephasing time at different initial spin polarizations. As predicted, the spin dephasing time increases with increasing temperature at small spin polarization but decreases at large spin polarization. The theoretical calculations also nicely reproduced the experimental data. The hot-electron temperatures in the calculation were taken from the experiment [Fig. 12(b)]. The effective magnetic field from the Hartree-Fock term has been measured by Zhang *et al.* from the sign switch of the Kerr signal and the phase reversal of Larmor precessions with a bias voltage in a GaAs heterostructure [57]. Korn *et al.* [58] also estimated the average effect by applying an external magnetic field in the Faraday configuration, as shown in Fig. 13(a) for the same sample reported above [55, 56]. They compared the spin dephasing times of both large and small spin polarizations as function of external magnetic field. Due to the effective magnetic field from the Hartree-Fock term, the spin relaxation times are different under small external magnetic field but become identical

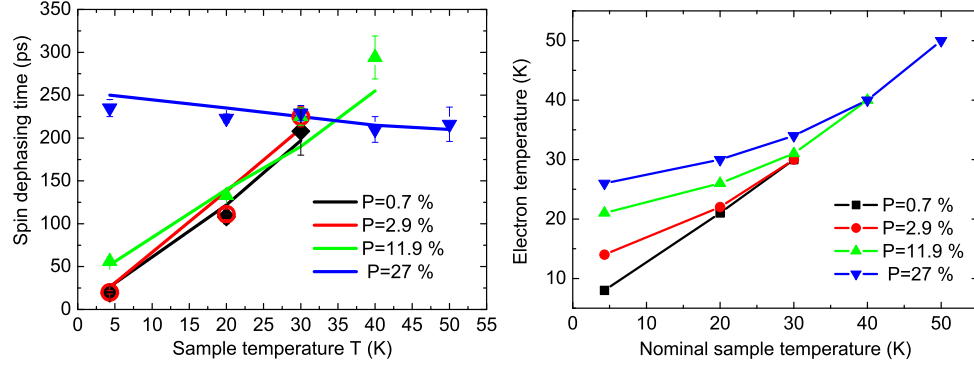


Figure 12: (a) Spin dephasing time as a function of sample temperature, for different initial spin polarizations. The measured data points are represented by solid points, while the calculated data are represented by lines of the same colour. (b) Electron temperature determined from intensity-dependent photoilluminance measurements as a function of the nominal sample temperature, for different pump beam fluence and initial spin polarization, under experimental conditions corresponding to the measurements shown in (a). The measured data points are represented by solid points, while the curves serve as guide to the eye. From Stich et al. [56].

when the magnetic field becomes large enough. From the merging point, they estimated the mean value of the effective magnetic field is below 0.4 T. They further showed that this effective magnetic field from the Hartree-Fock term cannot be compensated by the external magnetic field, because it does not break the time-reversal symmetry and is therefore not a genuine magnetic field, as said above. This can be seen from Fig. 13(b) that the spin relaxation time at large spin polarization shows identical external magnetic field dependences when the magnetic field is parallel or antiparallel to the growth direction.

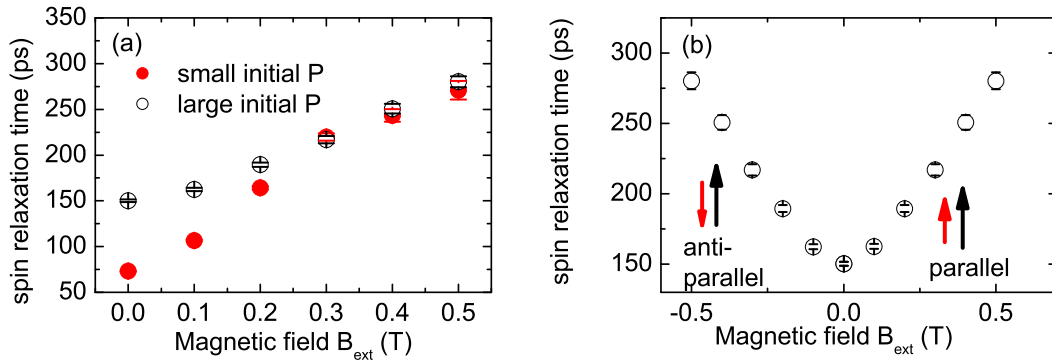


Figure 13: (a) Spin dephasing times as a function of an external magnetic field perpendicular to the quantum well plane for small and large initial spin polarization. (b) Same as (a) for large initial spin polarization and both polarities of the external magnetic field. From Korn et al.[58]

We now turn to discuss the spin relaxation/dephasing far away from the equilibrium. In fact, the spin relaxation/dephasing of high spin polarization addressed above is one of the cases far away from the equilibrium. Another case is the spin dynamics in the presence of a high in-plane electric field. The spin dynamics in the presence of a high in-plane electric field

was first studied by Weng *et al.* [22] in GaAs quantum well with only the lowest subband by solving the kinetic spin Bloch equations. To avoid the “runaway” effect [59], the electric field was calculated upto 1 kV/cm. Then Weng and Wu further introduced the second subband into the model and the in-plane electric field was increased upto 3 kV/cm [28]. Zhang *et al.* included L valley and the electric field was further increased upto 7 kV/cm [32]. The effect of in-plane electric field to the spin relaxation in system with strain was investigated by Jiang and Wu [51]. Zhou *et al.* also investigated the electric-field effect at low lattice temperatures [29].

The in-plane electric field leads to two effects: i) It shifts the center-of-mass of electrons to $\mathbf{k}_d = m^* \mathbf{v}_d = m^* \mu \mathbf{E}$ with μ representing the mobility, which further induces an effective magnetic field via the D’yakonov-Perel’ term [22]. ii) The in-plane electric field also leads to the hot-electron effect [60]. The first effect induces a spin precession even in the absence of any external magnetic field and the spin precession frequency changes with the direction of the electric field in the presence of an external magnetic field [22, 32]. The second effect enhances both the inhomogeneous broadening and the scattering, two competing effects leading to rich electric-field dependence of the spin relaxation/dephasing and thus spin manipulation [22, 28, 32, 51, 29, 33, 41].

Finally we address some issues of which the kinetic-spin-Bloch-equation approach gives qualitatively different predictions from those widely used in the literature. These issues include the Bir-Aronov-Pikus mechanism, the Elliot-Yafet mechanism and some density/temperature dependences of the spin relaxation/dephasing time.

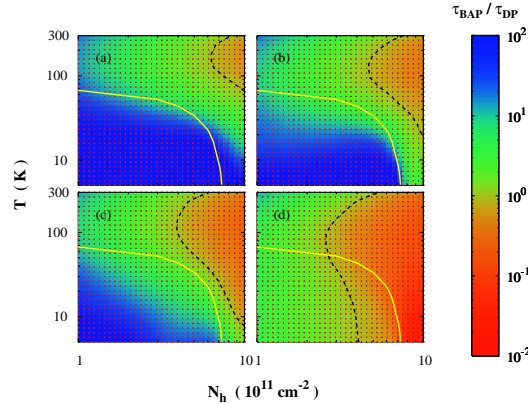


Figure 14: Ratio of the spin relaxation time due to the Bir-Aronov-Pikus mechanism to that due to the D’yakonov-Perel’ mechanism, $\tau_{\text{BAP}}/\tau_{\text{DP}}$, as function of temperature and hole density with (a) $N_i = 0$, $N_{ex} = 10^{11} \text{ cm}^{-2}$; (b) $N_i = 0$, $N_{ex} = 10^9 \text{ cm}^{-2}$; (c) $N_i = N_h$, $N_{ex} = 10^{11} \text{ cm}^{-2}$; (d) $N_i = N_h$, $N_{ex} = 10^9 \text{ cm}^{-2}$. The black dashed curves indicate the cases satisfying $\tau_{\text{BAP}}/\tau_{\text{DP}} = 1$. Note the smaller the ratio $\tau_{\text{BAP}}/\tau_{\text{DP}}$ is, the more important the Bir-Aronov-Pikus mechanism becomes. The yellow solid curves indicate the cases satisfying $\partial_{\mu_h}[N_{\text{LH}(1)} + N_{\text{HH}(2)}]/\partial_{\mu_h} N_h = 0.1$. In the regime above the yellow curve the multi-hole-subband effect becomes significant. From Zhou *et al.* [36].

It has long been believed in the literature that for electron relaxation/dephasing, the Bir-Aronov-Pikus mechanism is dominant at low temperature in p -type samples and has important contribution to intrinsic sample with high photo-excitation [61, 62, 63, 64, 65, 66, 67]. These conclusion was made based on the single-particle Fermi golden rule. Zhou and We re-examined the problem using the kinetic-spin-Bloch-equation approach [30]. They pointed out

that the Pauli blocking was overlooked in the Fermi Golden rule approach. When electrons are in the non-degenerate limit, the results calculated from the Fermi Golden rule approach are valid. However, at low temperature, electrons can be degenerate and the Pauli blocking becomes very important. As a result, the previous approaches always overestimated the importance of the Bir-Aronov-Pikus mechanism at low temperature. Moreover, the previous single-particle theories underestimated the contribution of the D'yakonov-Perel' mechanism by neglecting the Coulomb scattering. Both made the Bir-Aronov-Pikus mechanism dominate the spin relaxation/dephasing at low temperature. Later, Zhou *et al.* performed a thorough investigation of electron spin relaxation in *p*-type (001) GaAs quantum wells by varying impurity, hole and photo-excited electron densities over a wide range of values [36], under the idea that very high impurity density and very low photo-excited electron density may effectively suppress the importance of the D'yakonov-Perel' mechanism and the Pauli blocking. Then the relative importance of the Bir-Aronov-Pikus and D'yakonov-Perel' mechanisms may be reversed. This indeed happens as shown in the phase-diagram-like picture in Fig. 14 where the relative importance of the Bir-Aronov-Pikus and D'yakonov-Perel' mechanisms is plotted as function of hole density and temperature at low and high impurity densities and photo-excitation densities. For the situation of high hole density they even included multi-hole subbands as well as the light hole band. It is interesting to see from the figures that at relatively high photo-excitations, the Bir-Aronov-Pikus mechanism becomes more important than the D'yakonov-Perel' mechanism only at high hole densities and high temperatures (around hole Fermi temperature) when the impurity is very low [zero in Fig. 14(a)]. Impurities can suppress the D'yakonov-Perel' mechanism and hence enhance the relative importance of the Bir-Aronov-Pikus mechanism. As a result, the temperature regime is extended, ranging from the hole Fermi temperature to the electron Fermi temperature for high hole density. When the photo-excitation is weak so that the Pauli blocking is less important, the temperature regime where the Bir-Aronov-Pikus mechanism is important becomes wider compared to the high excitation case. In particular, if the impurity density is high enough and the photo-excitation is so low that the electron Fermi temperature is below the lowest temperature of the investigation, the Bir-Aronov-Pikus mechanism can dominate the whole temperature regime of the investigation at sufficiently high hole density, as shown in Fig. 14(d). The corresponding spin relaxation times of each mechanism under high or low impurity and photo-excitation densities are demonstrated in Fig. 14. They also discussed the density dependences of spin relaxation with some intriguing properties related to the high hole subbands [36]. The predicted Pauli-blocking effect in the Bir-Aronov-Pikus mechanism has been partially demonstrated experimentally by Yang *et al.* [68] They showed by increasing the pumping density, the temperature dependence of the spin dephasing time deviates from the one from the Bir-Aronov-Pikus mechanism and the peaks at high excitations agree well with those predicted by Zhou and Wu [30].

Another widely accepted but incorrect conclusion is related to the Elliot-Yafet mechanism. It is widely accepted in the literature that the Elliot-Yafet mechanism dominates spin relaxation in *n*-type bulk III-V semiconductor at low temperature, while the D'yakonov-Perel' mechanism is important at high temperature [69, 5, 7, 8, 70]. Jiang and Wu pointed out that the previous understanding are based on the formula that can only be used in the nondegenerate limit. Moreover, the momentum relaxation rates are calculated via the approximated formula for mobility [69]. By performing an accurate calculation via the kinetic-spin-Bloch-equation approach, they showed that the Elliot-Yafet mechanism is *not* important in III-V semiconductors, including even the narrow-band InAs and InSb [41]. Therefore, the D'yakonov-Perel' mechanism is the only spin relaxation mechanism for *n*-type III-V semiconductors in metallic regime.

Jiang and We have further predicted a peak in the density dependence of the spin relax-

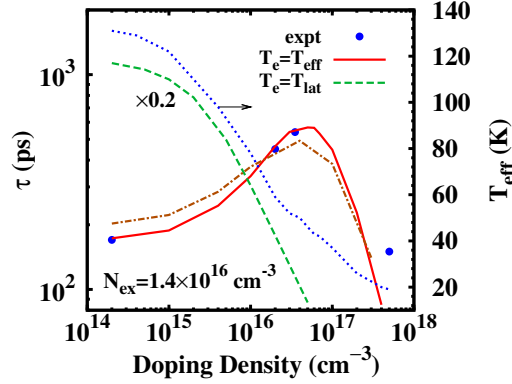


Figure 15: Electron spin relaxation times from the calculation via the kinetic-spin-Bloch-equation approach (red solid curve) and from the experiment[72] (blue •) as function of the doping density. The green dashed curve shows the results without hot-electron effect. The hot-electron temperature used in the computation is plotted as the blue dotted curve (Note the scale is on the right hand side of the frame). The chain curve is the calculated spin relaxation time with a fixed hot-electron temperature 80 K, and $N_{ex} = 6 \times 10^{15} \text{ cm}^{-3}$. From Shen [73].

ation/dephasing time in n -type III-V semiconductors where the spin relaxation/dephasing is limited by the D'yakonov-Perel' mechanism [41]. Previously, the nonmonotonic density dependence of spin lifetime was observed in low-temperature ($T \lesssim 5 \text{ K}$) experiments, where the localized electrons play a crucial role and the electron system is in the insulating regime or around the metal-insulator transition point [71]. Jiang and Wu found, for the first time, that the spin lifetime in *metallic* regime is also *nonmonotonic*. Moreover, they pointed out that it is a *universal* behavior for *all* bulk III-V semiconductors at *all* temperature where the peak is located at $T_F \sim T$ with T_F being the electron Fermi temperature. The underlying physics for the nonmonotonic density dependence in metallic regime can be understood as following: In the nondegenerate regime, as the distribution is the Boltzmann one, the density dependence of the inhomogeneous broadening is marginal. However, the scattering increases with the density. Consequently the spin relaxation/dephasing time increases with the density. However, in the degenerate regime, due to the Fermi distribution, the inhomogeneous broadening increases with the density much faster than the scattering does. As a result, the spin relaxation/dephasing time decreases with the density. Similar behavior was also found in two-dimensional system [31, 37], where the underlying physics is similar. The predicted peak was later observed by Krauß *et al.* [72] as shown in Fig. 15 where theoretical calculation based on the kinetic spin Bloch equations nicely reproduced the experimental data by Shen [73].

0.4.3 Spin diffusion/transport

By solving the kinetic spin Bloch equations together with the Poisson equation self-consistently, one is able to obtain all the transport properties such as the mobility, charge diffusion length and spin diffusion/injection length without any fitting parameter. It was first pointed out by Weng and Wu [23] that the drift-diffusion equation approach is inadequate in accounting for the spin diffusion/transport. It is important to include the off-diagonal term between oppo-

site spin bands $\rho_{\mathbf{k}\uparrow\downarrow}$ in studying the spin diffusion/transport. With this term, electron spin precesses along the diffusion and therefore $\mathbf{k} \cdot \nabla_{\mathbf{r}} \rho_{\mathbf{k}}(\mathbf{r}, t)$ in the diffusion terms $\dot{\rho}_{\mathbf{k}}(\mathbf{r}, t)|_{\text{dif}}$ provides an additional inhomogeneous broadening. With this additional inhomogeneous broadening, any scattering, including the Coulomb scattering, can cause an irreversible spin relaxation/dephasing [23]. Unlike the spin precession in the time domain where the inhomogeneous broadening is determined by the effective magnetic field from the D'yakonov-Perel' term, $\mathbf{h}(\mathbf{k})$, in spin diffusion and transport it is determined by

$$\Omega_{\mathbf{k}} = |g\mu_B \mathbf{B} + \mathbf{h}(\mathbf{k})|/k_x, \quad (142)$$

provided the diffusion is along the x -axis [42]. Here the magnetic field is in the Voigt configuration. Therefore, even in the absence of the D'yakonov-Perel' term $\mathbf{h}(\mathbf{k})$, the magnetic field *alone* can provide an inhomogeneous broadening and leads to the spin relaxation/dephasing in spin diffusion and transport. This was first pointed out by Weng and Wu back to 2002 [23] and has been realized experimentally by Appelbaum et al. in bulk silicon [74, 75], where there is no D'yakonov-Perel' spin-orbit coupling due to the center inversion symmetry. Zhang and Wu further investigated the spin diffusion and transport in symmetric Si/SiGe quantum wells [43].

When $B = 0$ but the D'yakonov-Perel' term is present, then the inhomogeneous broadening for spin diffusion and transport is determined by $\Omega_{\mathbf{k}} = \mathbf{h}(\mathbf{k})/k_x$. In (001) GaAs quantum well where the D'yakonov-Perel' term is determined by the Dresselhaus term [76], the average of $\Omega_{\mathbf{k}}$ reads $\langle \Omega_{\mathbf{k}} \rangle = C(\langle k_y^2 \rangle - \langle k_z^2 \rangle, 0, 0)$ with C being a constant. For electrons in quantum well, this value is not zero. Therefore, the spacial spin oscillation due to the Dresselhaus effective magnetic field survives even at high temperature when the scattering is strong. This effect was first predicted by Weng and Wu by showing a spin pulse can oscillate along the diffusion in the absence of the magnetic field at very high temperature [24]. Detailed studies were carried out later on this effect [25, 77, 42]. The spin oscillation without any applied magnetic field in the transient spin transport was later observed experimentally by Crooker and Smith in strained bulk system [78]. differing from the two-dimensional case, in bulk the average of $\Omega_{\mathbf{k}}$ from the Dresselhaus term is zero, since $\langle \Omega_{\mathbf{k}} \rangle = C(\langle k_y^2 \rangle - \langle k_z^2 \rangle, 0, 0) = 0$ due to the symmetry in the y - and z -directions. This is consistent with the experimental result that there is no spin oscillation for the system without stress. However, when the stress is applied, an additional spin-orbit coupling, namely the coupling of electron spins to the strain tensor, appears, which is linear in momentum [5]. This additional spin-orbit coupling also acts as an effective magnetic field. Therefore, once the stress is applied, one can observe spacial spin oscillation even when there is no applied magnetic field [78].

Cheng and Wu further developed a new numerical scheme to calculate the spin diffusion/transport in GaAs quantum wells with very high accuracy and speed [42]. It was discovered that due to the scattering, especially the Coulomb scattering, $T_2 = T_2^*$ is valid even in the spacial domain. This prediction remains yet to be verified experimentally. Moreover, as the inhomogeneous broadening in spin diffusion is determined by $|\mathbf{h}(\mathbf{k})|/k_x$ in the absence of magnetic field, the period of the spin oscillations along the x -axis is independent on the electric field perpendicular to the growth direction of the quantum well [42], which is different from the spin precession rate in the time domain [22]. This is consistent with the experimental findings by Beck *et al.* [79].

Cheng *et al.* applied the kinetic-spin-Bloch-equation approach to study the spin transport in the presence of competing Dresselhaus and Rashba fields [80]. When the Dresselhaus and Bychkov-Rashba [10] terms are both important in semiconductor quantum well, the total effective magnetic field can be highly anisotropic and spin dynamics is also highly anisotropic

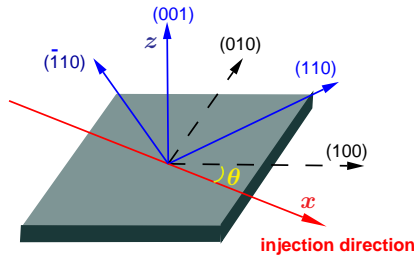


Figure 16: Schematic of the different directions considered for the spin polarizations [(110), $(\bar{1}10)$ and (001)-axes] and spin diffusion/injection (x -axis). From Cheng *et al.* [80].

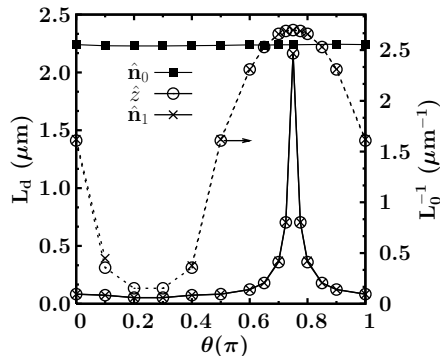


Figure 17: Spin diffusion length L_d (solid curves) and the inverse of the spin oscillation period L_0^{-1} (dashed curves) for identical Dresselhaus and Rashba coupling strengths as functions of the injection direction for different spin polarization directions $\hat{\mathbf{n}}_0$, $\hat{\mathbf{z}}$ and $\hat{\mathbf{n}}_1$ ($\hat{\mathbf{n}}_1 = \hat{\mathbf{z}} \times \hat{\mathbf{n}}_0$, i.e., crystal direction $[\bar{1}10]$) at $T = 200$ K. It is noted that the scale of the spin oscillation period is on the right hand side of the frame. From Cheng *et al.* [80].

in regards to the spin polarization [81]. For some special polarization direction, the spin relaxation time is extremely large [81, 82, 83, 84]. For example, if the coefficients of the linear Dresselhaus and Bychkov-Rashba terms are equal to each other in (001) quantum well of small well width and the cubic Dresselhaus term is not important, the effective magnetic field is along the [110] direction for all electrons. For the spin components perpendicular to the [110] direction, this effective magnetic field flips the spin and leads to a finite spin relaxation/dephasing time. For spin along the [110] direction, this effective magnetic field can not flip it. Therefore, when the spin polarization is along the [110] direction, the Dresselhaus and Bychkov-Rashba terms can not cause any spin relaxation/dephasing. When the cubic Dresselhaus term is taken into account, the spin dephasing time for spin polarization along the [110] direction is finite but still much larger than other directions [85]. The anisotropy in the spin direction is also expected in spin diffusion and transport. When the Dresselhaus and Bychkov-Rashba terms are comparable, the spin injection length L_d for the spin polarization perpendicular to [110] direction is usually much shorter than that for the spin polarization along [110] direction. In the ideal case when there are only the linear Dresselhaus and Bychkov-Rashba terms with identical strengths, spin injection length for spin polarization parallel to the [110] direction becomes infinity [82, 83]. This effect has promoted Schliemann *et al.* to propose the nonballistic spin-field-effect transistor [83]. In such a transistor, a gate voltage is used to tune the strength of the Bychkov-Rashba term and therefore control the spin injection length. However, Cheng *et al.* pointed out that spin diffusion and transport

actually involve both the spin polarization and spin transport directions [80]. The latter has long been overlooked in the literature. In the kinetic-spin-Bloch-equation approach, this direction corresponds to the spacial gradient in the diffusion term $[\dot{\rho}_{\mathbf{k}}(\mathbf{r}, t)|_{\text{diff}}]$ and the electric field in the drifting term $[\dot{\rho}_{\mathbf{k}}(\mathbf{r}, t)|_{\text{dr}}]$. The importance of the spin transport direction has not been realized until Cheng *et al.* pointed out that the spin transport is highly anisotropic not only in the sense of the spin polarization direction but also in the spin transport direction when the Dresselhaus and Bychkov-Rashba effective magnetic fields are comparable [80]. They even predicted that in (001) GaAs quantum well with identical linear Dresselhaus and Bychkov-Rashba coupling strengths, the spin injection along $[\bar{1}10]$ or $[110]$ [86] can be infinite *regardless of* the direction of the spin polarization. This can be easily seen from the inhomogeneous broadening Eq. (142) which well defines the spin diffusion/transport properties. For the spin diffusion/transport in a (001) GaAs quantum well with identical Dresselhaus and Bychkov-Rashba strengths (the schematic is shown in Fig. 16 with the transport direction chosen along the x -axis), the inhomogeneous broadening is given by [80]

$$\begin{aligned} \Omega_{\mathbf{k}} = & \left\{ 2\beta \left(\sin\left(\theta - \frac{\pi}{4}\right) + \cos\left(\theta - \frac{\pi}{4}\right) \frac{k_y}{k_x} \right) \hat{\mathbf{n}}_0 \right. \\ & \left. + \gamma \left(\frac{k_x^2 - k_y^2}{2} \sin 2\theta + k_x k_y \cos 2\theta \right) \left(\frac{k_y}{k_x}, -1, 0 \right) \right\}, \end{aligned} \quad (143)$$

with θ being the angle between the spin transport direction (x -axis) and $[001]$ crystal direction. It can be splitted into two parts: the zeroth-order term (on k) which is always along the same direction of $\hat{\mathbf{n}}_0$ and the second-order term which comes from the cubic Dresselhaus term. If the cubic Dresselhaus term is omitted, the effective magnetic fields for all \mathbf{k} states align along $\hat{\mathbf{n}}_0$ (crystal $[110]$) direction. Therefore, if the spin polarization is along $\hat{\mathbf{n}}_0$, there is no spin relaxation even in the presence of scattering since there is no spin precession. Nevertheless, it is interesting to see from Eq. (143) that when $\theta = 3\pi/4$, i.e., the spin transport is along the $[\bar{1}10]$ direction, $\Omega_{\mathbf{k}} = 2m^*\beta\hat{\mathbf{n}}_0$ is independent on \mathbf{k} if the cubic Dresselhaus term is neglected. Therefore, in this special spin transport direction, there is no inhomogeneous broadening in the spin transport for *any* spin polarization. The spin injection length is therefore infinite regardless of the direction of spin polarization. This result is highly counterintuitive, considering that the spin relaxation times for the spin components perpendicular to the effective magnetic field are finite in the spacial uniform system. The surprisingly contradictory results, i.e., the finite spin relaxation/dephasing time versus the infinite spin injection length, are due to the difference in the inhomogeneous broadening in spacial uniform and non-uniform systems. For genuine situation, due to the presence of the cubic term, the spin injection length is still finite and the maximum spin injection length does not happen at the identical Dresselhaus and Bychkov-Rashba coupling strengths, but shifted by a small amount due to the cubic term [80]. However, there is strong anisotropy in regards to the spin polarization and spin injection direction, as shown in Fig. 17. This predication has not yet been realized experimentally. However, very recent experimental findings on spin helix [13, 14] have provide strong evidence to support this predication [87].

Now we turn to the problem of spin grating. Transient spin grating, whose spin polarization varies periodically in real space, is excited optically by two non-collinear coherent light beams with orthogonal linear polarization [88, 89, 90, 14]. Transient spin grating technique can be used to study the spin transport since it can directly probe the decay rate of nonuniform spin distributions. Spin diffusion coefficient D_s can be obtained from the transient spin grating experiments [88, 89, 90]. In the literature, the drift-diffusion model was employed to extract D_s from the experimental data. With the drift-diffusion model,

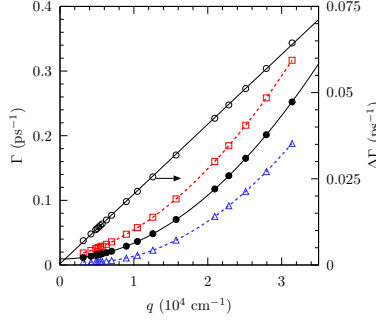


Figure 18: $\Gamma = (\Gamma_+ + \Gamma_-)/2$ and $\Delta\Gamma = (\Gamma_+ - \Gamma_-)/2$ vs. q at $T = 295$ K. Open boxes/triangles are the relaxation rates $\Gamma_{+/-}$ calculated from the full kinetic spin Bloch equations. Filled/open circles represent Γ and $\Delta\Gamma$ respectively. Noted that the scale for $\Delta\Gamma$ is on the right hand side of the frame. The solid curves are the fitting to Γ and $\Delta\Gamma$ respectively. The dashed curves are guide to eyes. From Weng *et al.* [91].

the transient spin grating was predicted to decay exponentially with time with a decay rate of $\Gamma_q = D_s q^2 + 1/\tau_s$, where q is the wavevector of the spin grating and τ_s is the spin relaxation time [88, 89]. However, this result is not accurate since it neglects the spin precession which plays an important role in spin transport as first pointed out by Weng and Wu [22]. Indeed, experimental results show that the decay of transient spin grating takes a double-exponential form instead of single exponential one [88, 90, 14]. Also the relation which relates the spin injection length with the spin diffusion coefficient D_s and the spin relaxation time τ_s $L_s = 2\sqrt{D_s \tau_s}$ from the drift-diffusion model should be checked. In fact, if this relaxation is correct, the above prediction of infinite spin injection length at certain spin injection direction for any spin polarization in the presence of identical Dresselhaus and Bychkov-Rashba coupling strengths cannot be correct.

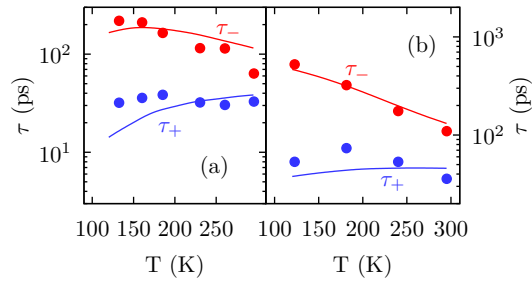


Figure 19: Spin relaxation times τ_{\pm} vs. temperature for (a) high-mobility sample with $q = 0.58 \times 10^4$ cm $^{-1}$ and (b) low-mobility sample with $q = 0.69 \times 10^4$ cm $^{-1}$. The dots are the experiment data from Ref. [90]. From Weng *et al.* [91].

Weng *et al.* studied this problem from the kinetic-spin-Bloch-equation approach [91]. By first solving the kinetic spin Bloch equations analytically by including only the elastic scattering, *i.e.*, the electron-impurity scattering, they showed that the transient spin grating should decay double exponentially with two decay rates Γ_{\pm} . In fact, none of the rates is quadratic in q . However, the average of them reads [91]

$$\Gamma = (\Gamma_+ + \Gamma_-)/2 = Dq^2 + 1/\tau'_s \quad (144)$$

with $1/\tau'_s = (1/\tau_s + 1/\tau_{s1})/2$, which differs from the current widely used formula by replacing

the spin decay rate by the average of the in- and out-of-plane relaxation rates. The difference of these two decay rates is a linear function of the wavevector q when q is relatively large:

$$\Delta\Gamma = cq + d, \quad (145)$$

with c and d being two constants. The steady-state spin injection length L_s and spin precession period L_0 are then [91]

$$L_s = 2D_s / \sqrt{|c^2 - 4D_s(1/\tau'_s - d)|}, \quad (146)$$

$$L_0 = 2D_s / c. \quad (147)$$

They further showed that the above relations Eqs. (144) and (145) are valid even including the inelastic electron-electron and electron phonon scatterings by solving the full kinetic spin Bloch equations, as shown in Fig. 18. A good agreement with the experimental data [90] of double exponential decays $\tau_{\pm} = \Gamma_{\pm}^{-1}$ are shown in Fig. 19. Finally it was shown that the infinite spin injection length predicted by Cheng *et al.* in the presence of identical Dresselhaus and Bychkov-Rashba coupling strengths [80] addressed above can exactly be obtained from Eq. (146) as in that special case $\tau'_s = \tau_s$, $d = 0$ and $c = 2\sqrt{D_s/\tau_s}$ [91]. However, $\sqrt{D_s\tau_s}$ always remains finite unless $\tau_s = \infty$. Therefore, Eqs. (144-147) give the correct way to extract the spin injection length from the spin grating measurement.

Acknowledgements

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